1 **Parameterization of the Solar Fluxes over Mountain Surfaces for Application to Climate**

2 **Models**

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Abstract

On the basis of 3-D Monte Carlo photon tracing simulations, we have developed a parameterization of solar fluxes over mountain surfaces by means of the multiple linear regression analysis associated with topographic information, including the following parameters as independent variables: elevation, solar incident angle, sky view factor, and terrain configuration factor. The regression equation for the direct flux can explain more than 98% of the variation in which the solar incident angle is the dominant factor, except when the sun is very low or at zenith. About 60% of the variation in the diffuse flux is predicted by the regression equation in which the mean elevation, the sky view factor, and the solar incident angle are key factors. The terrain-reflected fluxes, proportional to the surface albedo, are well correlated with the terrain configuration factor with more than 80% of the variation that can be explained. The coupled fluxes involve intricate interactions and the regression analysis is less satisfactory in cases of low albedo values. However, over high-albedo snow/ice surfaces, the terrain configuration factor in correlation analysis becomes most dominant, leading to a significant improvement in regression performance. Using a region over the Sierra Nevada Mountains as a test bed, the preceding regression parameterizations have been specifically developed so that the fluxes evaluated from the 3-D Monte Carlo photon tracing program over intense topography can be used as a perturbation term to correct those computed from the plane-parallel counterpart, commonly used in regional climate models and GCMs.

1. Introduction

The spatial and temporal distribution of the surface solar radiation is critical to many processes including snowmelt, soil moistening, photosynthesis, and evapotranspiration [Geiger, 1965; Bonan, 2002; Gu *et al.*, 2002; Müller and Scherer, 2005]. It results from interactions between the direct and diffuse solar beams and the surface. Because of the complexity of spatial orientation and optical properties of the surface, quantifying these interactions and hence surface fluxes has been very difficult.

A number of models with a varying degree of sophistication have been developed to evaluate surface solar fluxes over mountainous areas [Hay and McKay, 1985; Duguay, 1993]. In these models, the direct solar flux was calculated by employing the correction based on the cosine of solar incident angle, and considering the shadow effect caused by nearby mountains [e.g., Olyphant, 1986]. The diffuse solar flux has been modeled as being proportional to the sky view factor, which is the portion of sky dome visible to the target [Dozier and Frew, 1990]. However, as pointed out by Dubayah and Rich [1995], this approach assumes that diffuse radiation is isotropic. Wang *et al.* [2005] found that model performance can be significantly improved if the anisotropic nature of diffuse radiation is taken into account. Factors contributing to the terrain-reflected flux have generally not been considered explicitly, but are approximated by assuming a first-order reflection between surrounding terrain and the target surface [e.g., Dubayah *et al.*, 1990]. However, this

1 approximation does not consider the effect of multiple reflections, which are particularly important
2 over snow surfaces and steep valleys [Dozier, 1980]. Miesch *et al.* [1999] further investigated this
3 issue using a Monte Carlo approach over a 2-D rugged terrain to calculate surface solar fluxes, and
4 concluded that a 10% error can occur if the terrain effect is not accounted for.

5 *Chen et al.* [2006] developed a 3-D Monte Carlo radiative transfer model to calculate solar
6 fluxes exactly over mountains in a clear sky atmosphere and found that the reflected radiation can
7 account for 3% of the total surface fluxes during the winter time with a high surface albedo. They
8 also showed differences in the surface solar flux ranging from 5 to 20 W/m² between mountainous
9 and flat surfaces with the same mean elevation. *Liou et al.* [2007] utilized this 3-D model to
10 evaluate the surface fluxes over the Tibetan Plateau and illustrated that anomalies of the surface
11 solar flux with reference to a flat surface can be as large as 600 W/m² in shaded areas. The
12 domain-average solar surface flux over a regional model scale (e.g., 50×50 km²) with intense
13 topography can deviate from the smoothed surface by 10–50 W/m².

14 The 3-D Monte Carlo photon tracing simulation for radiative transfer involving mountains is
15 computationally expensive, and its direct implementation in a climate model would not be practical.
16 Parameterization of surface solar fluxes based on sub-grid-scale topographic characteristics is
17 therefore necessary. Parameterizations using topographic information have been developed to
18 calculate solar radiation over mountainous surfaces [Dubayah *et al.*, 1990; Essery and Marks, 2007].

1 However, these parameterizations are based on conventional 1-D radiative transfer models, where
2 multiple reflection effects are not accounted for. Moreover, these models neglect the anisotropic
3 nature of diffuse solar radiation.

4 In this study, we developed a parameterization of surface solar fluxes based on the results
5 from a 3-D Monte Carlo simulation using the intense topography of the Sierra Nevada Mountain as
6 our test bed. Employing topographic parameters as independent variables, multiple linear regression
7 analysis can be carried out to evaluate surface solar fluxes. Given the information about topography,
8 a more accurate and physically-based surface solar flux can be predicted from the resulting simple
9 linear regression equations. In current regional climate models and GCMs, conventional 1-D
10 radiative transfer schemes assuming a flat surface are typically used to compute solar fluxes. For
11 this reason, we develop a parameterization that predicts flux deviations over mountains from the
12 flat surface counterpart.

13 This paper is organized as follows: A brief introduction to the 3-D Monte Carlo photon tracing
14 program with the new surface configuration is given in Section 2. The various categories of
15 downward solar fluxes to be predicted, and several relevant topographic parameters to be used as
16 independent variables, such as the solar incident angle, the sky view factor, and the terrain
17 configuration factor are described in Section 3. In Section 4 we present the results of the multiple
18 linear regression for various solar flux components. In addition, we show the impact of topographic

variables on flux variations and their relative importance. Conclusions are given in Section 5.

2. A 3-D Radiative Transfer Model for Surface Solar Flux Calculations

The Monte Carlo approach used in this study generally follows the model developed by *Chen et al.* [2006], except for the land surface configuration. Therefore, only a brief description is given here. In the current approach, the atmospheric domain is divided into finite cuboid cells such that the optical properties are homogeneous within each cell. Photons entering the top of the atmosphere (TOA) are defined by unit irradiance, and their direction is given by the position of the sun. The optical path length l , which a photon travels from the TOA to its first encounter with an air molecule, or between two encounters with an air molecule, is determined by Beer's law: $l = -\ln \xi$, where ξ is a random number uniformly distributed within the interval $[0, 1]$. When a photon reaches the point of encounter, the probability it is absorbed or scattered is dependent on the single-scattering albedo, the ratio of the scattering cross-section to the extinction cross-section. In the absorption case, the photon is terminated. However, if scattering occurs, the photon is deflected in a direction determined by the mean scattering phase function of the cell, which is a function of the incoming and outgoing directions characterized by μ (cosine of the zenith angle, θ) and φ , the azimuth angle. Once the photon hits the ground, the energy of the reflected photon is equal to the product of the energy of incoming photon and the surface albedo. To a good approximation, the

land surface can be taken as Lambertian for flux calculations, that is, the reflected flux is isotropic. The direction of the reflected photon is determined by $\mu' = \cos \theta' = \sqrt{\xi_1}$ and $\varphi' = 2\pi\xi_2$, where ξ_1 and ξ_2 are random numbers within the interval $[0,1]$; θ' and φ' are the zenith and azimuth angles relative to the inclined surface, respectively. Each photon is traced until it is absorbed by the atmosphere or leaves the TOA. The flux at each cell facet can be obtained by tallying the total weights of the photons crossing the cell's boundary.

In the development of a 3-D Monte Carlo photon tracing model, *Chen et al.* [2006] adopted a stair-like form to represent mountain surfaces. This approximation is computationally efficient and convenient for programming implementation. However, such surfaces can result in more reflection events than those on realistic inclined surfaces. For example, for a point on a horizontal facet very close to a vertical facet of the stair-like form, almost half of the sky dome is obstructed by the vertical facet. When a photon hits this point and is reflected, there is about 50% probability that this photon will then hit the vertical facet, leading to the occurrence of another reflection event. This effect can trap more energy at the corner between a horizontal and a vertical facet, producing a substantial surface flux. However, the presence of vertical cliffs is not very common for realistic land surfaces. Furthermore, because a slope is treated as a combination of a horizontal and a vertical facet in this approximation, the modeled surface area would be larger than the real surface area, resulting in underestimation of the surface radiative flux in the whole domain. Although the

preceding two effects might compensate each other in flux calculations over a large domain, it could produce significant errors locally.

In view of the above, we have developed a more realistic surface model using the following approach. In each pixel the surface is constructed by means of triangles based on the elevation data available from the digital elevation model (DEM). Using the method introduced by *Jenness* [2004], we connected the pixel's center point to the center points of eight surrounding pixels to generate eight 3-D triangles. Subsequently, the portions of those triangles residing within the pixel's boundaries are considered as the "real" topography of the pixel in the model. These triangles are also used to estimate the surface area of each pixel. As also shown by *Jenness* [2004], the area estimated from this method is close to that calculated with more sophisticated methods using the pixel number more than 250.

For the absorption by air molecules, we followed the correlated k -distribution approach [*Fu and Liou*, 1992; *Gu et al.*, 2003]. This allows for effective incorporation of line absorption in the 3-D Monte Carlo photon tracing program covering the entire solar spectrum. The absorbing gases included in broadband calculations are H₂O, CO₂, O₃, O₂, CH₄, N₂O, CFCs, CO, O₂O₂, O₂N₂, and NO₂ [*Gu et al.*, 2006]. The solar spectral region was divided into 6 bands: 0.2–0.7, 0.7–1.3, 1.3–1.9, 1.9–2.5, 2.5–3.5, and 3.5–5.0 μm , corresponding to the k -values of 10, 8, 12, 7, 12, and 5, respectively. Since we are focused on producing a parameterization of the sub-grid-scale terrain

1 effect on surface solar fluxes, Rayleigh scattering is the only scattering mechanism included in the
2 Monte Carlo simulations. We also assumed a climatological mean of the mid-latitude atmospheric
3 profile. In the conclusion, we touch upon how variations in atmospheric gaseous profiles, aerosols,
4 and clouds might be incorporated.

5 To test the accuracy of Monte Carlo photon tracing, 4×10^6 photons were first assigned to a
6 $20 \times 20 \text{ km}^2$ computational domain with a $1 \times 1 \text{ km}^2$ resolution, so that approximately 10^4 photons are
7 targeted toward each surface pixel. We then increased the number of photons by a factor of 10 (i.e.,
8 to 4×10^7), and found that the resulting relative difference in the mean surface flux is less than 0.3%.
9 Thus, use of 10^4 photons per pixel should suffice to achieve accurate statistics and, this photon
10 density is employed throughout this study.

11 As the experiment test bed, we selected a rugged area of the Sierra Nevada Mountains in the
12 Western United States, composed of a variety of elevations and slopes (Figure 1). The surface
13 topography with a 1 km resolution was taken from the HYDRO1k geographic database available
14 from the U. S. Geological Survey's (USGS) National Center for Earth Resources Observation and
15 Science (EROS) data center. This area is divided into eighty $20 \times 20 \text{ km}^2$ domains to represent the
16 general terrain characteristics and topographical variations. Because the model design is cyclic at
17 the domain boundary, photons exiting the western boundary will re-enter the domain at the eastern
18 boundary. As a result, areas with low elevation near the eastern boundary could be shadowed by

mountains at the western side. In order to reduce this edge effect, only the topographic information and surface radiative fluxes in the central $10 \times 10 \text{ km}^2$ areas were used for parameterization. The purpose of this study is to investigate the mountain effect; therefore, a uniform surface albedo value is used in Monte Carlo simulations, although we illustrate the sensitivity of fluxes involving terrain reflection to surface albedo by performing an additional set of Monte Carlo simulations with elevated—but still uniform—surface albedo.

3. Variables for Multiple Linear Regression Analysis

As shown in previous studies [e.g. *Chen et al.*, 2006], the direct, diffuse, and reflected solar fluxes are each controlled by different topographic variables. Thus, we have divided the surface solar fluxes into categories and perform regressions for each flux category separately. As illustrated in Figure 2, the solar radiative fluxes can be categorized into five components according to the path of the photons that make them up: (1) Direct flux, denoted as F_{dir} , is composed of photons hitting the ground directly from the sun without encountering scattering or reflection; (2) Diffuse flux, denoted as F_{dif} , contains photons experiencing single or multiple scattering by air molecules, but never encountering surface reflection; (3) Direct-reflected flux, denoted as F_{rdir} , is comprised of unscattered photons reflected by nearby terrain; (4) Diffuse-reflected flux, denoted as F_{rdif} , is similar to direct-reflected flux, except that the photon is first scattered by air molecules and then

reflected by nearby terrain; (5) Coupled flux, denoted as F_{coup} , represents photons that, after being reflected by the surface, encounter scattering, or one or more additional surface reflections. It is clear that the direct and diffuse fluxes are independent of surface albedo. Since F_{rdir} and F_{rdif} contain photons that have been reflected once, they are simply proportional to albedo. Only the coupled flux has nonlinear relationship with surface albedo. It is also noted that there will be no F_{rdir} and F_{rdif} on a flat surface. In this study, the solar radiative flux received by mountain surfaces is defined as the total solar energy reaching the ground divided by the flat surface area instead of the mountain area. This definition is selected so that differences in the solar energy budget between mountainous and flat surfaces can be compared in a consistent manner.

In current climate models, conventional plane-parallel radiative transfer schemes have already developed to calculate solar fluxes on a flat surface, given that surface's elevation. Therefore, the purpose of the present parameterization effort is to produce deviations of the fluxes from those of a flat surface, given sub-grid-scale topographic information. More specifically, the results of the multiple linear regression analysis done in this study are deviations of the five components of downward solar surface fluxes described above from those of a flat surface with the same mean elevation. To prepare the independent variables for regression, we introduce the concept of the sky view factor and the terrain configuration factor, which are widely used in estimating the diffuse and reflected fluxes over mountainous areas, respectively.

The sky view factor V_d was originally defined as the ratio of the diffuse flux received by a point to that received by an unobstructed horizontal surface [Dubayah *et al.*, 1990]. However, because the geometry of diffuse radiation is dependent on the position of the sun and atmospheric conditions, calculating the sky view factor is computationally expensive and storage intensive. If one assumes the diffuse flux is isotropic, the sky view factor becomes the portion of the sky dome visible to the target point as shown in Figure 2. This term can be obtained by calculating the area of unobstructed hemisphere given by [Dozier and Frew, 1990]

$$V_d = \frac{1}{\pi} \int_0^{2\pi} \int_0^{H_\varphi} \sin \theta [\cos \theta \cos \theta_s + \sin \theta \sin \theta_s \cos(\varphi - \varphi_s)] d\theta d\varphi \quad (1)$$

$$\approx \frac{1}{2\pi} \int_0^{2\pi} [\cos \theta_s \sin^2 H_\varphi + \sin \theta_s \cos(\varphi - \varphi_s) \times (H_\varphi - \sin H_\varphi \cos H_\varphi)] d\varphi,$$

where θ_s is the slope, φ_s is the aspect of the slope, and H_φ is the horizon angle measured from the zenith down to the local horizon at the orientation direction φ , which can result either from “self-shadowing” by the slope itself or from nearby mountains.

The terrain configuration factor C_t , which is defined as the area of surrounding mountains visible to the target point, also shown in Figure 2, can be written as [Dozier and Frew, 1990]

$$C_t = \frac{1}{\pi} \int_0^{2\pi} \int_{H_\varphi}^{\psi_\varphi} \sin \theta [\cos \theta \cos \theta_s + \sin \theta \sin \theta_s \cos(\varphi - \varphi_s)] d\theta d\varphi \quad (2)$$

$$\approx \frac{1 + \cos \theta_s}{2} - V_d,$$

where ψ_φ is the angle of the ray parallel to the slope defined by

$$\tan \psi_\varphi = \frac{-1}{\tan \theta_s \cos(\varphi - \varphi_s)}. \quad (3)$$

On an unobstructed infinite slope, the direct solar flux is proportional to the cosine of the solar incident angle θ_i , which is defined as the angle between the direction of incident ray and the normal vector of the slope and can be expressed by

$$\cos \theta_i = \cos \theta_s \cos \theta_0 + \sin \theta_s \sin \theta_0 \cos(\varphi_s - \varphi_0), \quad (4)$$

where θ_0 and φ_0 are the solar zenith and aspect angles, respectively. Defining $\mu_0 = \cos \theta_0$, and $\mu_i = \cos \theta_i$, the mean μ_i of the whole domain can be calculated by taking the spatial average of Eq. (4):

$$\begin{aligned} \langle \mu_i \rangle &= \mu_0 \langle \cos \theta_s \rangle + \sin \theta_0 \langle \sin \theta_s \cos(\varphi_s - \varphi_0) \rangle \\ &= \mu_0 \langle \cos \theta_s \rangle + \sin \theta_0 (\cos \varphi_0 \langle \sin \theta_s \cos \varphi_s \rangle + \sin \varphi_0 \langle \sin \theta_s \sin \varphi_s \rangle), \end{aligned} \quad (5)$$

where angle brackets denote spatial mean. Therefore, given the position of the sun, the average of μ_i for the whole domain can be determined if the means of $\cos \theta_s$, $\sin \theta_s \cos \varphi_s$, and $\sin \theta_s \sin \varphi_s$ are given.

In addition to the mean values for the sky view factor $\langle V_d \rangle$, the terrain configuration factor $\langle C_t \rangle$, and the cosine of the solar incident angle $\langle \mu_i \rangle$, conventional topographic parameters for each 10×10 km² domain, such as the mean elevation $\langle h \rangle$ and slope $\langle \theta_s \rangle$, are used. We also employed their standard deviation ($\sigma(h)$, $\sigma(\theta_s)$, $\sigma(V_d)$, $\sigma(C_t)$, and $\sigma(\mu_i)$) and skewness ($\gamma(h)$, $\gamma(\theta_s)$, $\gamma(V_d)$, $\gamma(C_t)$, and $\gamma(\mu_i)$) in the multiple linear regression analysis. In this manner, 15 independent variables are available for the regression analysis. The magnitudes of the independent variables vary substantially and the units associated with them may be different. Therefore, it is desirable to non-dimensionalize the variables prior to performing the regression analysis. All independent variables are processed by first removing their means and then normalizing them by their respective standard deviations.

It should be noted that these variables are not necessarily orthogonal to each other. Since it is desirable to produce a parameterization with as few independent variables as possible, independent variables highly correlated to one another must be reduced to a single variable. To accomplish this, we first calculate correlation coefficients between each successive pair of variables. When the correlation coefficient between two variables is larger than 0.9, one of them will be removed. It turns out that some variables are indeed highly correlated. As implied by Eq. (2), the correlation coefficient between $\langle V_d \rangle$ and $\langle C_t \rangle$ is very close to -1. Also, $\langle \theta_s \rangle$ is highly correlated with $\langle C_t \rangle$ and therefore it is anti-correlated with $\langle V_d \rangle$. Furthermore, $\langle \theta_s \rangle$, $\langle V_d \rangle$, $\langle C_t \rangle$, and $\langle \mu_i \rangle$ are highly correlated with their standard deviations $\sigma(\theta_s)$, $\sigma(V_d)$, $\sigma(C_t)$, and $\sigma(\mu_i)$, respectively. As a result, the independent variables for the regression of F_{dir} and F_{dif} are $\langle h \rangle$, $\sigma(h)$, $\chi(h)$, $\chi(\theta_s)$, $\langle V_d \rangle$, $\chi(V_d)$, $\langle \mu_i \rangle$, and $\chi(\mu_i)$, while $\langle V_d \rangle$ and $\chi(V_d)$ are replaced by $\langle C_t \rangle$ and $\chi(C_t)$ for F_{dir} , F_{dif} , and F_{coup} because C_t is originally designed for reflected-related fluxes. Also, note that when μ_0 is equal to 1, $\langle \mu_i \rangle$ is highly correlated to $\langle \theta_s \rangle$ since when the solar zenith angle is zero, θ_i is equal to θ_s according to Eq. (4). Therefore, when the sun is at the zenith, $\langle \mu_i \rangle$ and $\chi(\mu_i)$ will be removed from the regression analysis.

4. Results of Parameterization and Discussion

We choose seven different solar zenith angles (SZA) in this study, including $\mu_0 = 0.1, 0.25, 0.4, 0.55, 0.7, 0.85$, and 1. For each SZA (except $\mu_0 = 1$), four different solar azimuth angles ($0^\circ, 90^\circ$,

180°, and 270°) are applied for each of the 80 domains. Therefore, there are 320 samples for each SZA (but only 80 samples for $\mu_0 = 1$) to perform multiple linear regression. To obtain the simplest possible regression equation for each flux component, if the regression coefficients do not differ significantly from zero by using the t -test at the 90% confidence level, the corresponding independent variables are removed from regression equations. All flux components are in unit of W/m^2 . Since all independent variables are normalized, their associated regression coefficients represent the flux anomaly magnitude when the variable is increased by one standard deviation.

a. Direct flux: The regression equation for deviation of the direct flux can be expressed by

$$F_{\text{dir}} = a_1 + b_{11}\langle\mu_i\rangle + b_{12}\langle V_d\rangle, \quad (6)$$

where a_1 is the intercept term, b_{11} and b_{12} are regression coefficients. Values of a and b for each μ_0 are listed in Table 1. The intercept term represents the mean deviations between mountainous and flat surfaces. When the sun's position is very low, the shading effect leads to a negative deviation on the order of 1.5 W/m^2 , though the mean deviation generally becomes insignificant when the sun is close to the zenith. The cosine of the solar incident angle is the dominant factor in direct flux calculations. A one standard deviation increase in $\langle\mu_i\rangle$ can lead to $17\text{-}36 \text{ W/m}^2$ increases in the direct solar flux, except when the sun is at zenith, when $\langle\mu_i\rangle$ variations barely lead to anomalies in F_{dir} . Except when the sun is close to the horizon, the incoming flux becomes smaller in association with larger sky view factors.

To provide information about the explanatory power of the regression model, the left panels of Figure 3 compare the direct fluxes between the 3-D model and regression results for three values of μ_0 . The coefficients of multiple determination R^2 , listed in Table 2, are larger than 0.98 except for $\mu_0 = 0.1$ and 1, revealing that Eq. (6) can predict the direct flux quite accurately. Very small R^2 value when $\mu_0 = 1$ indicates that the direct flux is almost independent of $\langle\mu_i\rangle$ and $\langle V_d\rangle$. When the sun is at the zenith, there will be no shading effect at all; in this case, the variation of $\langle\mu_i\rangle$ is very small so that the direct flux cannot be explained by these two parameters. Even if all 8 independent variables mentioned in the end of Section 3 are used in the regression, R^2 is still smaller than 0.07, implying that the variation is rather random. Nevertheless, as shown in Figure 3, variation in the direct flux for $\mu_0 = 1$ is about 2 W/m², much smaller than variations in other cases, which are on the order of 100 W/m².

b. Diffuse flux: The regression equation for deviation of the diffuse flux can be expressed by

$$F_{\text{dif}} = a_2 + b_{21}\langle h \rangle + b_{22}\langle \mu_i \rangle + b_{23}\langle V_d \rangle + b_{24}\chi(h). \quad (7)$$

Values of the intercept and regression coefficients are also listed in Table 1. The most significant factor determining deviation of the diffuse flux is the mean elevation, with which it is positively correlated. To understand the relationship between F_{dif} and $\langle h \rangle$, we calculate the vertical distribution of the downward diffuse flux without topographic effect. As shown in Figure 4, the curves of downward diffuse fluxes are concave, and the curvature generally increases with increasing

elevation. Therefore, for two domains with the same $\sigma(h)$ and $\gamma(h)$ but different $\langle h \rangle$, domain averaged deviations of the diffuse flux will be larger for the domain with larger $\langle h \rangle$. Furthermore, since the curves are concave, the domain with a positive $\gamma(h)$ will receive less diffuse fluxes than a domain with the same mean elevation but negative $\gamma(h)$. This will lead to anti-correlation between F_{dif} and $\gamma(h)$, as shown in Table 1. However, there is an inflection point on the curve for $\mu_0 = 0.1$, leading to smaller regression coefficients for both $\langle h \rangle$ and $\gamma(h)$.

F_{dif} also increases with increasing $\langle \mu_i \rangle$, illustrating the anisotropic nature of diffuse flux. Since the maxima of the phase function for Rayleigh scattering resides at the forward and backward directions for an unpolarized light beam, the majority of scattered photons move in the same direction as the direct solar beam, resulting in a positive correlation between F_{dif} and $\langle \mu_i \rangle$. Even though the diffuse flux is anisotropic, the results determined from multiple linear regression display that F_{dif} is largely proportional to the mean sky view factor based on the isotropic assumption, which has been used by many previous studies [e.g. *Dozier and Frew*, 1990]. The regression coefficients of $\langle V_d \rangle$ decrease as μ_0 increases, indicating that the shading effect becomes less important when the sun is close to the zenith. On the other hand, the regression coefficients for $\langle \mu_i \rangle$ increase with increasing μ_0 . These two effects compensate each other and lead to an almost constant R^2 on the order of 0.6 when $\langle \mu_i \rangle$ is less than 1.0, as listed in Table 2. Comparisons between the regression and simulation results are also shown in the right panels of Figure 3. The range of

deviations in the diffuse flux is approximately 5 W/m^2 , which is $\sim 15\%$ of the total.

c. Direct- and diffuse-reflected fluxes: The regression equations for deviations of the direct-reflected and diffuse-reflected fluxes can be expressed, respectively, by

$$F_{\text{rdir}} = a_3 + b_{31}\langle C_t \rangle + b_{32}\langle h \rangle + b_{33}\gamma(h), \quad (8)$$

$$F_{\text{rdif}} = a_4 + b_{41}\langle C_t \rangle + b_{42}\langle h \rangle + b_{43}\gamma(h), \quad (9)$$

where the intercept terms and regression coefficients are listed in Table 3. Eqs. (8) and (9) can explain variation in the reflected fluxes quite well. Also noted in Table 2, the R^2 values for F_{rdir} and F_{rdif} are generally about 0.85 and 0.9, respectively, except when the sun is close to the horizon. Comparisons between the regression and simulation results are displayed in Figure 5. The intercept terms, which represent the mean responses, are always positive because these two fluxes are zero on a flat surface. The magnitudes of intercepts and regression coefficients for F_{rdir} increase with increasing μ_0 in view of the fact that F_{rdir} is positively correlated to F_{dir} , which in turn is proportional to μ_0 . The dominant independent variable in the regression model for F_{rdir} and F_{rdif} is the mean terrain configuration factor $\langle C_t \rangle$. The factor $\langle h \rangle$ also has a positive impact on F_{rdir} and F_{rdif} because the reflected photons encounter less absorption or scattering at a higher altitude before hitting the ground again. The negative regression coefficients for $\gamma(h)$ are a result of the fact that the extinction coefficient, which is proportional to the air density, increases as the elevation decreases.

The variation in F_{rdir} is $\sim 1\text{--}8 \text{ W/m}^2$, depending on μ_0 , whereas the variation in F_{rdif} is ~ 0.3

W/m², as shown in Figure 5. It should be noted that F_{rdir} and F_{rdif} are functions of the surface albedo. Because F_{rdir} and F_{rdif} fluxes are composed of photons that undergo one reflection only, they are directly proportional to the surface albedo. In Eqs. (8) and (9), the regression coefficients were derived from 3-D Monte Carlo simulations for a surface albedo of 0.1. To obtain F_{rdir} and F_{rdif} with different albedo values, we can linearly scale these coefficients according to the new albedo value. For example, over the snow-covered surface with an albedo of 0.7, variations in F_{rdir} and F_{rdif} would be 7–56 and 2.1 W/m² (a factor of 7), respectively.

d. Coupled flux: Deviation of the coupled flux can be expressed by the following regression equation:

$$F_{\text{coup}} = a_5 + b_{51}\langle\mu_i\rangle + b_{52}\langle C_t\rangle + b_{53}\langle h\rangle, \quad (10)$$

where the intercept term and the regression coefficients are listed in Table 4. From the values of R^2 listed in Table 2, we find that the explanation power of Eq. (10) is generally less than those for other flux components. The intercept terms are always negative. This is because the coupled flux is approximately the backscattering of the total energy reflected to the atmosphere. Because some reflected photons are intercepted by nearby mountains, the total amount of the coupled flux over rugged areas will be less than that over a flat surface. Since the incoming direct and diffuse fluxes are positively correlated with $\langle\mu_i\rangle$, the total amount of the reflected solar energy should also be proportional to $\langle\mu_i\rangle$, leading to positive values of b_{51} . Additionally, F_{rdir} and F_{rdif} are independent of

$\langle\mu_i\rangle$ so that the energy reflected back to the atmosphere, which can be estimated by subtracting F_{rdir} and F_{rdif} from the total reflected energy, is also proportional to $\langle\mu_i\rangle$. The negative regression coefficients for $\langle C_i\rangle$ indicate that the real physical mechanism determining F_{coup} is the V_d , not C_t , since the backscattering effect dominates. Comparisons between the regression and simulation results are displayed in Figure 6, and it shows that variations in coupled fluxes are within 0.5 W/m^2 for this surface albedo value.

The coupled flux is a nonlinear function of the surface albedo. For this reason, we have carried out an additional regression analysis using an albedo of 0.7. The intercept terms and regression coefficients are also listed in Table 4. As shown by R^2 in Table 2, the new regression equation can explain variation in the coupled flux much better than that with an albedo of 0.1. As shown in Figure 6, the associated variations in the coupled flux can reach 8 W/m^2 , much larger than that on a less-reflective surface. All regression coefficients increase as a result of more energy being reflected by the surface. The most important factor becomes $\langle C_t\rangle$, and the regression coefficients for $\langle C_t\rangle$ and the intercept terms are all positive. These two facts reveal that as surface albedo increases, the effect due to multiple reflection becomes more dominant than that due to backscattering by the atmosphere. The impact of $\langle h\rangle$ on F_{coup} is the same as on F_{rdir} and F_{rdif} , though this effect is not significant when albedo is 0.1. For practical application to climate models, several suitable albedo values can be selected for parameterization of the coupled fluxes, and regression parameters can be

interpolated for intermediate albedo values.

5. Conclusions

We have developed a new 3-D radiative transfer model for mountains on the basis of a Monte Carlo photon tracing program from which the most essential radiative components are parameterized for potential application to climate models. A rugged area in the Sierra Nevada Mountains in the Western United States was chosen as a test bed, and deviations of the domain averaged solar fluxes were analyzed. Several topographic parameters were introduced as independent variables for the multiple linear regression analysis, including the solar incident angle, elevation, slope, the sky view factor, the terrain configuration factor, as well as the standard deviation and skewness of these parameters.

Using multiple linear regression analysis, deviations of the five components of surface fluxes are expressed by a linear combination of topographic parameters. Variation in the direct flux is much larger than other components and generally larger than 100 W/m^2 . Except when the sun is at the zenith or close to the horizon, we show that the mean solar incident angle and mean sky view factor in the regression equation can explain more than 98% of the variation, and the corresponding root mean square errors (RMSE) are generally less than 3 W/m^2 . Although the explanation power is almost zero for the direct flux with the sun at the zenith, its variation is less than 2 W/m^2 , implying

1 that the impact is of less importance. However, the RMSE can reach 10 W/m^2 when the sun is very
2 close to the horizon. For the diffuse flux, about 60% of the variation can be predicted by the mean
3 elevation, the mean solar incident angle, the mean sky view factor, and the skewness of elevation.
4 The variation in the diffuse flux is about 5 W/m^2 , so it leads to an RMSE less than 0.5 W/m^2 ,
5 smaller than that for the direct flux. The dominant factor for determining the direct-reflected and
6 diffuse-reflected fluxes is the terrain configuration factor with R^2 about 0.85 and 0.9, respectively.
7 The magnitudes of the variation and RMSE for the direct-reflected flux are respectively 1-8 and
8 $0.08\text{-}0.5 \text{ W/m}^2$, depending on the position of the sun. Variation and RMSE for the diffuse-reflected
9 flux are consistently about 0.3 and 0.01 W/m^2 , respectively. Although they are dependent on the
10 surface albedo, their variations can be linearly scaled by using an albedo ratio factor (e.g, $0.7/0.1$).
11 The coupled flux component is more complicated because it contains photons experiencing multiple
12 scattering and reflection. For this reason, the explanation power of the regression equation for a low
13 albedo of 0.1 is less than those for other flux components. However, because the magnitude of the
14 variation in the coupled flux is only about 0.5 W/m^2 , the resulting RMSE are less than 0.07 W/m^2 .
15 For a scenario with an albedo of 0.7, the performance of the regression equation are better with R^2
16 larger than 0.7. Variation in the coupled flux could reach 8 W/m^2 due to the increase in reflected
17 energy over highly reflective surface, and the RMSE becomes 0.6 W/m^2 . In this case, the terrain
18 configuration factor becomes the dominant variable. Combining the regression equations for all five

1 components, the departure of the solar flux over the rugged area from that of a flat surface can be
2 imposed as a correction to the flat-surface flux calculated by the conventional plane-parallel
3 radiative transfer schemes. The performance of the regression equations for the coupled flux is less
4 satisfactory; however, the error magnitude is so small that it does not affect the total flux calculation.
5 In this manner, much more accurate surface energy budgets can be obtained for the coupled
6 land-atmosphere system where intense topography is present.

7 Although a mean atmospheric profile is used for all simulations, the present parameterization
8 can be applied to different atmospheric compositions, since variation in trace gas concentrations
9 only alters the absorption coefficient in association with the number of absorbed photons. However,
10 the number and direction of scattered photons remain unchanged. Consequently, we can linearly
11 scale the regression coefficients based on new absorption coefficients to obtain the corrections for
12 the surface solar fluxes.

13 We may also apply the present regression-based parameterization to a background aerosol
14 condition coupled with Rayleigh scattering, but accounting for variation in the aerosol loading (e.g.,
15 using aerosol optical depth) would require further numerical experiments. When clouds are present,
16 the geometry of scattered photons will be significantly changed depending on their position and
17 optical and microphysical properties (optical depth and cloud particle size). Thus, the regression
18 equations for the diffuse, diffuse-reflected, and coupled fluxes would need significant modifications.

1 The direct and direct-reflected fluxes, however, would require only linear scaling with respect to the
2 change in cloud extinction coefficient. In summary, resolving the cloud and aerosol problems within
3 the context of the parameterization of surface radiative fluxes over mountains is a challenging task,
4 and requires additional in-depth theoretical and numerical investigations.

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1 **Table Captions**

2 **Table 1.** Intercepts and Regression Coefficients for Eqs. (6) and (7).

3 **Table 2.** R^2 for Eqs. (6), (7), (8), (9), and (10).

4 **Table 3.** Intercepts and Regression Coefficients for Eqs. (8) and (9).

5 **Table 4.** Intercepts and Regression Coefficients for Eq. (10) for Two Surface Albedos of 0.1 and

6 0.7.

Figure Captions

Figure 1. Maps of the elevation (left, unit in meter) and slope (right, unit in degree) in the Sierra Nevada Mountains area.

Figure 2. A schematic representation of flux components received by the target on an inclined surface in a mountainous area: (1) direct flux, (2) diffuse flux, (3) direct-reflected flux, (4) diffuse-reflected flux, and (5) coupled flux. The solar incident angle θ_i , the sky view factor V_d and the terrain configuration factor C_t are also shown.

Figure 3. Comparison between the results from the Monte Carlo photon tracing simulation (real values) and the multiple linear regression analysis (predicted values) for the direct flux (left panels) and the diffuse flux (right panels).

Figure 4. Vertical distribution of the downward diffuse fluxes over a flat surface with elevation of zero for $\mu_0 = 0.1, 0.4$, and 1.0 .

Figure 5. As in Figure 3, but for the direct-reflected flux (left panels) and diffuse-reflected flux (right panels).

Figure 6. As in Figure 3, but for the coupled flux with a surface albedo of 0.1 (left panels) and a surface albedo of 0.7 (right panels).

1 **Table 1.** Intercepts and Regression Coefficients for Eqs. (6) and (7).

μ_0	Direct Flux			Diffuse Flux				
	a_1	$\langle \mu_i \rangle$	$\langle V_d \rangle$	a_2	$\langle h \rangle$	$\langle \mu_i \rangle$	$\langle V_d \rangle$	$\gamma(h)$
0.10	-1.629	18.90	2.201	-0.157	0.204	0.046	0.253	-0.089
0.25	-0.399	33.89	-0.425	-0.060	0.341	0.110	0.285	-0.130
0.40	-0.041	37.29	-2.748	-0.013	0.389	0.185	0.270	-0.145
0.55	-0.081	36.25	-4.081	0.002	0.401	0.247	0.258	-0.156
0.70	-0.098	32.39	-5.420	0.010	0.395	0.273	0.233	-0.152
0.85	-0.058	25.04	-6.705	0.004	0.382	0.245	0.199	-0.152
1.00	-0.095	0.058	-0.030	0.009	0.324	0	0.244	-0.134

1 **Table 2.** R^2 for Eqs. (6), (7), (8), (9), and (10).

μ_0	F_{dir}	F_{dif}	F_{rdir}	F_{rdif}	F_{coup}	
					alb=0.1	alb=0.7
0.10	0.756	0.609	0.447	0.898	0.476	0.466
0.25	0.987	0.611	0.819	0.903	0.673	0.714
0.40	0.999	0.609	0.855	0.903	0.645	0.757
0.55	0.998	0.626	0.861	0.901	0.543	0.767
0.70	0.996	0.630	0.860	0.901	0.423	0.768
0.85	0.991	0.620	0.858	0.900	0.265	0.767
1.00	0.006	0.572	0.861	0.895	0.182	0.780

1 **Table 3.** Intercepts and Regression Coefficients for Eqs. (8) and (9).

μ_0	Direct-Reflected Flux				Diffuse-Reflected Flux			
	a_3	$\langle C_i \rangle$	$\langle h \rangle$	$\gamma(h)$	a_4	$\langle C_i \rangle$	$\langle h \rangle$	$\gamma(h)$
0.10	0.170	0.069	0.016	-0.006	0.052	0.025	0.003	-0.002
0.25	0.569	0.271	0.055	-0.024	0.081	0.039	0.003	-0.004
0.40	0.999	0.475	0.100	-0.044	0.091	0.044	0.003	-0.004
0.55	1.439	0.679	0.149	-0.064	0.097	0.047	0.003	-0.004
0.70	1.882	0.883	0.197	-0.085	0.100	0.049	0.003	-0.004
0.85	2.330	1.086	0.247	-0.105	0.102	0.050	0.003	-0.005
1.00	2.783	1.298	0.292	-0.126	0.104	0.051	0.003	-0.005

1 **Table 4.** Intercepts and Regression Coefficients for Eq. (10) for Two Surface Albedos of 0.1 and
2 0.7.

μ_0	Coupled Flux (Albedo = 0.1)				Coupled Flux (Albedo = 0.7)			
	a_5	$\langle\mu_i\rangle$	$\langle C_t\rangle$	$\langle h\rangle$	a_5	$\langle\mu_i\rangle$	$\langle C_t\rangle$	$\langle h\rangle$
0.10	-0.004	0.014	-0.002	0.002	0.136	0.100	0.081	0.017
0.25	-0.019	0.038	-0.001	-0.002	0.390	0.265	0.302	0.023
0.40	-0.041	0.046	-0.004	-0.002	0.640	0.311	0.518	0.040
0.55	-0.064	0.046	-0.008	-0.002	0.869	0.302	0.720	0.077
0.70	-0.088	0.040	-0.014	0.002	1.114	0.256	0.917	0.108
0.85	-0.111	0.027	-0.020	0.003	1.356	0.167	1.105	0.139
1.00	-0.128	0	-0.032	0.002	1.604	0	1.250	0.154

Figure 1. Maps of elevation (left, unit in meter) and slope (right, unit in degree) in the Sierra Nevada Mountains area.

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