Data-based paleoclimate stochastic model

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Introduction

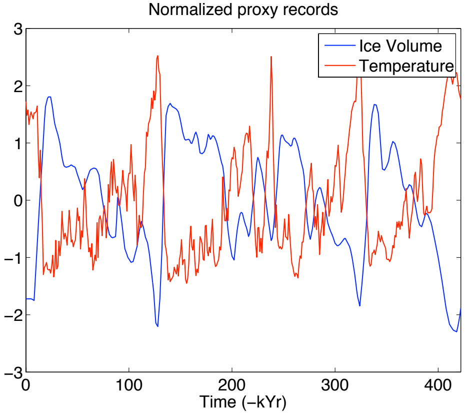
We apply the empirical mode reduction (EMR) methodology (Kravtsov et al., 2005) to construct a hierarchy of dynamic, stochastically forced models for the analysis and simulation of paleoclimate data. We apply this methodology to a multivariate dataset consisting of Vostok ice-core and marine-core time series, representing proxy records for temperature and global ice-volume, respectively. In addition to residual stochastic forcing, the model is externally forced by orbital periodic forcing. The model's fit to the proxy records is verified by checking the probability density functions (PDFs) and autocorrelations, of the paleodata and of the reduced model's simulations. An analytical study of the reduced models suggests a role for stochastically forced internal variability, in addition to the periodic orbital forcing.

Data

- **Marine core** (proxy for ice volume ***V***):
Bassinot et al., The astronomical theory of climate and the age of the Brunhes-Matuyama magnetic reversal, *Earth and Planetary Science Letters*, 126 (1-3), 91-108, 1994.

- **Vostok core** (proxy for surface air temperature ***T***)

- Both records are interpolated to 1 Kyr resolution



Empirical Mode Reduction (EMR)

- Sometimes we have observational data but not a good physical model.

-We want models that are as simple as possible, but not any simpler!

Criteria for a good data-derived model:

- Capture statistics (histograms, correlations, spectra) and relevant dynamics: regimes, oscillations, etc.
- Deterministic dynamics easy to analyze analytically.
- Good noise estimates.
- Describes independent data.

General form of stochastic DE system

$$
\begin{aligned}
dx_i &= (\mathbf{x}^{\mathrm{T}}\mathbf{A}_i\mathbf{x} + \mathbf{b}_i^{(0)}\mathbf{x} + c_i^{(0)})\,dt + r_i^{(0)}\,dt,\\
dr_i^{(0)} &= \mathbf{b}_i^{(1)}[\mathbf{x}, \mathbf{r}^{(0)}]dt + r_i^{(1)}\,dt,\\
dr_i^{(1)} &= \mathbf{b}_i^{(2)}[\mathbf{x}, \mathbf{r}^{(0)}, \mathbf{r}^{(1)}]dt + r_i^{(2)}\,dt,\\
&\dots\\
dr_i^{(L)} &= \mathbf{b}_i^{(L)}[\mathbf{x}, \mathbf{r}^{(0)}, \mathbf{r}^{(1)}, \dots, \mathbf{r}^{(L)}]dt + dr_i^{(L+1)}
\end{aligned}
$$

- **matrices A_i**, **vectors b_i**, and **scalars c_i**. are estimated by **least squares**.
-**Multiple predictors are time series x_i**.
-**Predictant variables** are one-step **time differences** of predictors; **step = sampling interval Δt**.
-Multi-level modeling of **noise r_i**. to account serial correlations in the regression residuals.
- The number of levels is such that each of the last-level (L) regression residuals is "white" in time.
- Spatial (cross-channel) correlations of the last-level residuals are retained in subsequent regression-model simulations.
- Model nonlinearity is chosen to optimize the EMR performance (comparison with data).

Multiplicative periodic forcing:

- modify linear and constant terms on the main ("0") level to account for periodic forcing with period ***T***:

$$\mathbf{b}_i^{(0)} = \mathbf{b}_{i(0)}^{(0)} + \mathbf{b}_{i\mathrm{T}}^{(0)} f_p(t,T), c_i^{(0)} = c_{i(0)}^{(0)} + c_{i\mathrm{T}}^{(0)} f_p(t,T).$$

EMR for paleoclimate

Oscillatory feedbacks:

Ice-albedo: $\dot{T} \approx -V$

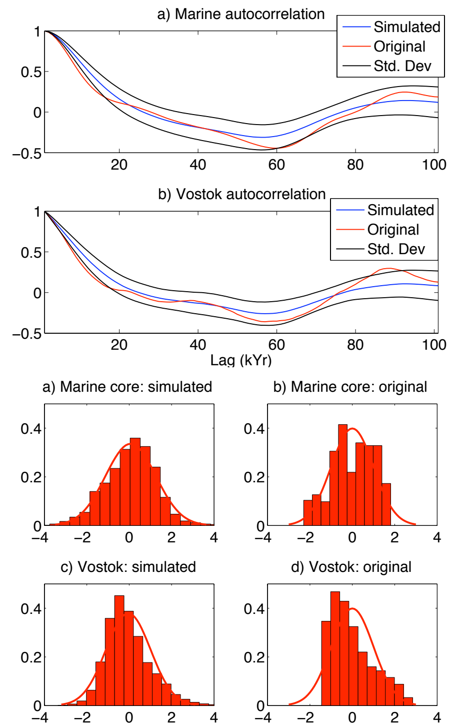
Temperature-precipitation: $\dot{V} \approx -T$

+ **Milankovitch forcing**:
obliquity (***41 kYr***), precession (***23,19 kYr***), eccentricity (***100 kYr***)

+ **nonlinearity**

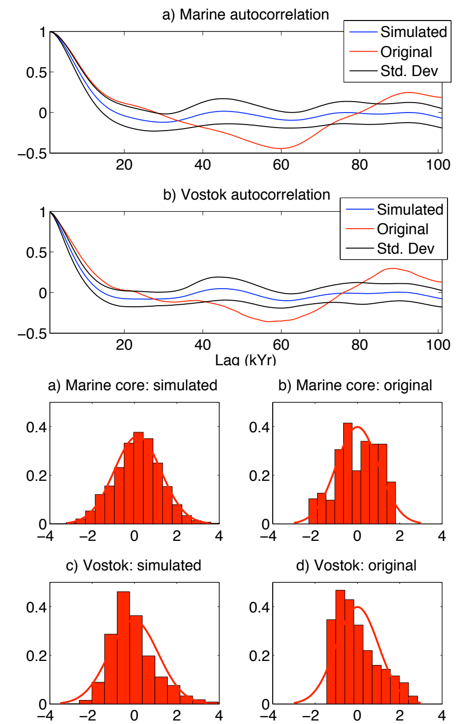
Results for **quadratic 3-L EMR** model for ***V*** and ***T***, **with quasi-periodic forcing** at 19, 23, 41, **and 100 Kyr**.

Statistics of original dataset and ensemble of 100 stochastic realizations:



Results for **quadratic 3-L EMR model for *V*** and ***T***, **with quasi-periodic forcing** at 19, 23, and 41Kyr, **but NONE at 100 Kyr**.

Characterized by damped oscillatory eigenmode with 93-Kyr period and 10-Kyr decay time.



References

1. Kondrashov, D., S. Kravtsov, A. W. Robertson, and M. Ghil, 2005: A hierarchy of data-based ENSO models. J. Climate, 18, 4425–4444.
2. Kondrashov, D., S. Kravtsov, and M. Ghil, 2006: Empirical mode reduction in a model of extratropical low-frequency variability. J. Atmos. Sci., (63), 1859–1877.
3. Kravtsov, S., D. Kondrashov, and M. Ghil, 2005: Multilevel regression modeling of nonlinear processes: Derivation and applications to climatic variability. J. Climate, 18, 4404–4424.

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