

# The Role of Oscillatory Modes in U.S. Business Cycles

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## Abstract

We apply the advanced time-and-frequency-domain method of singular spectrum analysis to study business cycle dynamics in a set of nine U.S. macroeconomic indicators. This method provides a robust way to identify and reconstruct shared oscillations, whether intermittent or modulated. We address the problem of spurious cycles generated by the use of detrending filters and present a Monte Carlo test to extract significant oscillations. Finally, we demonstrate that the behavior of the U.S. economy changes significantly between episodes of growth and recession; these variations cannot be generated by random shocks alone, in the absence of endogenous variability.

*Keywords:* Advanced spectral methods, Comovements, Frequency domain, Monte Carlo testing, Time domain

*JEL classification:* C15, C60, E32

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## 1. Introduction

Business cycles, their causes and characteristics have been extensively studied since the beginnings of modern economic theory. Research to characterize

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their regularities and stylized facts has, therewith, a long history (Burns and Mitchell, 1946; Kydland and Prescott, 1998). Dominated by a long-term upward drift, macroeconomic time series also exhibit smaller but still very important short-term fluctuations.

A number of approaches have been proposed to separate these fluctuations from the trend (Canova, 1998; Baxter and King, 1999); the Hodrick-Prescott filter (HP filter) is the most commonly used tool to do so (Hodrick and Prescott, 1997). Since there is no fundamental theory — and hence no generally accepted definition — of the trend, the resulting residuals have to be analyzed very critically, in order to avoid spurious results due merely to the detrending procedure (Nelson and Kang, 1981; Harvey and Jaeger, 1993; Cogley and Nason, 1995).

Business cycles are understood as comovements of the deviations from the trend in several distinct macroeconomic variables (Burns and Mitchell, 1946; Lucas, 1977). It is imperative, therefore, to analyze business cycle properties as a multivariate process. In effect, the U.S. National Bureau of Economic Research (NBER) proposes the following definition: “a recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales.”

The purpose of this paper is to apply the advanced methodology of *singular spectrum analysis (SSA)* — more specifically both univariate and *multivariate SSA (M-SSA)* — to the analysis of macroeconomic time series in order to help understand the origin and behavior of business cycles. Both methods rely on the classical Karhunen-Loève spectral decomposition of time series and random fields (Karhunen, 1946; Loève, 1945, 1978). Broomhead and King (1986a,b) proposed to use SSA and M-SSA in the context of nonlinear dynamics for the purpose of reconstructing the attractor of a system from measured time series, thus providing an extension and a more robust application of the Mañé-Takens idea of reconstructing dynamics from a single time series (Mañé, 1981; Takens, 1981; Sauer et al., 1991). Ghil, Vautard and associates (Vautard and Ghil, 1989; Ghil and Vautard, 1991; Vautard et al., 1992) noticed that SSA can be used as

a time-and-frequency domain method for time series analysis — independently from attractor reconstruction and including cases in which the latter may fail.

We rely here on M-SSA for the analysis of oscillatory modes and comovements of several time series that reflect the time evolution of a single economy. M-SSA combines two useful approaches of statistical analysis: (1) it determines — with the help of principal component analysis — major directions in the system’s phase space that are populated by the multivariate time series; and (2) it extracts major spectral components by using data-adaptive filters. In particular, M-SSA can separate distinct spectral components in a multivariate data set of limited length and in the presence of relatively high noise levels. In order to get reliable information about oscillatory modes, we perform exhaustive statistical tests by means of Monte Carlo SSA (Allen and Smith, 1996); these tests allow us to deal with the problem of spurious oscillations that was highlighted in the economic literature by Nelson and Kang (1981) and by Cogley and Nason (1995).

M-SSA has already proven its advantages in a variety of applications, such as climate dynamics, meteorology and oceanography, as well as the biomedical sciences. Ghil et al. (2002) provide an overview and a comprehensive set of references to both the theory and applications of SSA and M-SSA; free software for implementation is provided by the SSA-MTM Toolkit at <http://www.atmos.ucla.edu/tcd/ssa>. M-SSA has also shown its ability to reduce the effect of noise in order to help predict future exchange rates (Lisi and Medio, 1997). The present paper, however, goes beyond mere denoising and describes in much greater detail how to use M-SSA in order to quantify the properties of the underlying macroeconomic system.

The paper is organized as follows. In Section 2, we introduce the SSA and M-SSA methodology: starting from single-channel SSA, we discuss the extension to multi-channel time series. We summarize the properties of the methodology in terms of spectral decomposition, as well as of time-domain reconstruction. In Section 3, we apply single-channel SSA to the U.S. gross domestic product (GDP) and M-SSA to the full data set; the reliability of the results is then

discussed via Monte Carlo testing. Section 4 analyzes the cycle-to-cycle variability of the U.S. business cycles, and we draw conclusions about the underlying dynamics in Section 5.

## 2. Decomposition and reconstruction

### 2.1. Data and pre-processing

We apply M-SSA to U.S. macroeconomic data from the Bureau of Economic Analysis (BEA; see <http://www.bea.gov>). The nine variables analyzed are GDP, investment, employment rate, consumption, total wage, change in private inventories, price, exports, and imports; all monetary variables are in constant 2005 dollars, while the employment rate is in percentage points. The time series of these nine variables are available on a quarterly basis, and cover 52 years, from the first quarter of 1954 to the last of 2005.

We first remove the trend of each time series separately, by using the [Hodrick and Prescott \(1997\)](#) filter with the parameter value  $\lambda = 1600$ , as recommended by these authors for quarterly data. Employment is the only one of the nine variables that does not exhibit an upward trend, being bounded between 0 and 100%; still we detrend it, in order to remove periods that are much longer than 10 years and are thus not relevant to our analysis.

This approach to obtaining the raw–trend residuals provides a common basis for comparison with previous studies. As stated already in the Introduction, we are fully aware of the danger of obtaining spurious cyclical behavior due to inadequate detrending ([Nelson and Kang, 1981](#); [Harvey and Jaeger, 1993](#); [Cogley and Nason, 1995](#)), and will discuss in Section 3 the ability of Monte Carlo M-SSA to justify the results obtained in the present paper.

We nondimensionalize each time series by dividing the residuals by the trend — i.e., we concentrate on relative values — and then divide these relative values by their standard deviation, so that the resulting time series have unit standard deviation. Finally, we divide the normalized time series by  $(DM)^{1/2}$  — where  $D = 9$  is the number of variables (“channels”) and  $M = 24$  is the window

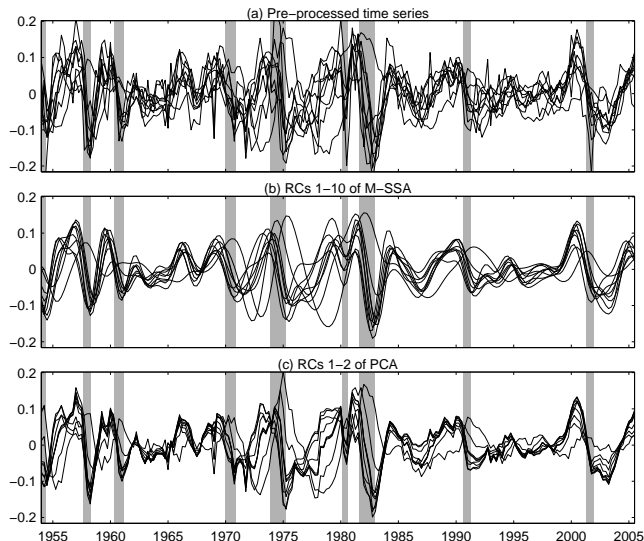


Figure 1: The nine time series of U.S. macroeconomic data used in this paper; raw data from the U.S. Bureau of Economic Analysis (BEA), 1954–2005. The figure illustrates the results of pre-processing and of applying either multivariate singular spectrum analysis (M-SSA) or principal component analysis (PCA); the shaded vertical bars in the three panels indicate NBER-defined recessions. (a) Detrended and standardized time series. (b,c) Reconstruction of the entire data set: (b) with the first 10 M-SSA components, using a window width of  $M = 24$  quarters; and (c) with the first two PCA components. Both reconstructions capture 75% of the total variance.

width — so that the sum of the partial variances equals one; see the next two subsections for justification and details.

Figure 1a shows the results of this pre-processing. The U.S. recessions, as defined by the NBER, are indicated by shaded vertical bars in this figure.

## 2.2. Singular spectrum analysis (SSA)

Before introducing the full multivariate SSA, we first discuss the *univariate version of SSA* and present the main properties and capabilities of this analysis method. The classical approach to describe cyclical behavior of a single time series is to decompose it into its spectral components by some version of Fourier analysis. This approach works well for fairly long time series  $\{x(t) : t = 1 \dots N\}$ , with  $N$  large, and relatively low noise levels; it works less well when  $N$  is small

and the noise is large, which is often the case in economic time series, as well as in geophysical ones.

Ghil, Vautard and several associates first proposed to apply the SSA methodology to handle the problem of describing cyclical behavior in short and noisy time series, for which standard methods derived from Fourier analysis do not work well (Vautard and Ghil, 1989; Ghil and Vautard, 1991; Vautard et al., 1992). This methodology provides insight into the unknown or partially known dynamics of the underlying system that has generated the time series. Applying SSA to reconstruct the entire attractor of a nonlinear dynamical system from limited data, as originally proposed by Broomhead and King (1986a), may fail, however, even in relatively simple cases (Mees et al., 1987; Vautard and Ghil, 1989).

The key idea of Vautard and Ghil (1989) and of Ghil and Vautard (1991) was to only reconstruct the “skeleton of the attractor”, i.e. the most robust, albeit unstable limit cycles embedded in it. Following Mañé (1981) and Takens (1981), the starting point of SSA is to embed the time series  $\{x(t) : t = 1 \dots N\}$  into an  $M$ -dimensional *phase space*  $\mathbf{X}$ , by using  $M$  lagged copies

$$\mathbf{x}(t) = (x(t), x(t+1), \dots, x(t+M-1)), \quad (1)$$

with  $t = 1 \dots N - M + 1$ .

The SSA procedure starts by calculating the principal directions of the attractor in this embedding phase space  $\mathbf{X}$  from the vector sequence  $\mathbf{x}(t)$ . The first step is then to compute the auto-covariance matrix  $\mathbf{C}$  of  $\mathbf{x}$ , whose elements  $c_{i,j}$  are given by

$$c_{i,j} = \frac{1}{N - |i-j|} \sum_{t=1}^{N-|i-j|} x(t)x(t+|i-j|). \quad (2)$$

Vautard and Ghil (1989) observed that the  $M \times M$  matrix  $\mathbf{C}$  has Toeplitz structure with constant diagonals: its entries  $c_{i,j}$  depend only on the lag  $|i-j|$ .

The eigenvalues  $\lambda_k$  and eigenvectors  $\boldsymbol{\rho}_k$  of  $\mathbf{C}$ ,  $k = 1 \dots M$ , are obtained by solving

$$\mathbf{C}\boldsymbol{\rho}_k = \lambda_k\boldsymbol{\rho}_k. \quad (3)$$

The eigenvectors, which are pairwise orthonormal, span a new coordinate system in the  $M$ -dimensional embedding space  $\mathbf{X}$ , and each eigenvalue  $\lambda_k$  indicates the variance of  $\mathbf{x}$  in the corresponding direction  $\boldsymbol{\rho}_k$ . This computation helps us find major directions in  $\mathbf{x}$  that carry a large variance, and therefore describe major components of the system’s dynamical behavior. Usually, the spectral decomposition of  $\mathbf{C}$  determines its directions of greatest variance successively, from the largest to the smallest eigenvalues, subject to the condition that each new direction be orthogonal to all the preceding ones.

By convention, the eigenvalues  $\{\lambda_k, k = 1 \dots M\}$  are arranged in descending order, thus representing the directions  $\boldsymbol{\rho}_k$  in  $\mathbf{x}$  in order of importance, from the largest to the smallest variance. In the so-called “scree diagram” plot of  $\lambda_k$  vs.  $k$ , one often looks for a clear break in the slope in order to distinguish “signal” from “noise.” Such a break, however, occurs mostly when the noise is actually white, with no temporal correlations at all. The signal-to-noise separation test has, therefore, to be modified in the presence of colored noise. We will discuss this problem in greater detail when introducing the appropriate significance test in Sec. 3.

Projecting the time series  $\mathbf{x}$  onto each eigenvector  $\boldsymbol{\rho}_k$  yields the corresponding principal component (PC)

$$A_k(t) = \sum_{j=1}^M x(t+j-1)\rho_k(j), \quad k = 1 \dots M. \quad (4)$$

Note that the sum above is not defined close to the end of the time series, where  $N - M \leq t \leq N$ . It is customary, therefore, to consider the PCs as defined for only  $N - M + 1$  indices, which could start at  $t = M$  and end at  $N$ , or start at  $t = 1$  but end at  $N - M + 1$ ; most commonly they are plotted centered for  $M/2 \leq t \leq N - M/2$ , with  $M$  even (Ghil et al., 2002).

We can now reconstruct, cf. Ghil and Vautard (1991) and Vautard et al. (1992), the part  $r_k(t)$  of the time series that is associated with a particular eigenvector,

$$r_k(t) = \frac{1}{M_t} \sum_{j=L_t}^{U_t} A_k(t-j+1)\rho_k(j), \quad k = 1 \dots M. \quad (5)$$

The values of the triplet of integers  $(M_t, L_t, U_t)$  for the central part of the time series,  $M \leq t \leq N - M + 1$ , are simply  $(M, 1, M)$ ; for either end they are given in Ghil et al. (2002). Thus the *reconstructed component* (RC)  $r_k(t)$  associated with the variance  $\lambda_k$  has a complete set of  $N$  indices; our confidence in its values, though, decreases as we approach either end of the time series, since fewer data points are averaged over the window  $M$  to obtain these RC values.

Given any subset  $\mathcal{K}$  of eigenelements  $\{(\lambda_k, \boldsymbol{\rho}_k) : k \in \mathcal{K}\}$ , we obtain the corresponding reconstruction  $r_{\mathcal{K}}(t)$  by summing the RCs  $r_k$  over  $k \in \mathcal{K}$ ,

$$r_{\mathcal{K}}(t) = \sum_{k \in \mathcal{K}} r_k(t). \quad (6)$$

Typical choices of  $\mathcal{K}$  may involve (i)  $\mathcal{K} = \{k : 1 \leq k \leq S\}$ , where  $S$  is the *statistical dimension* of the time series, cf. Vautard and Ghil (1989), i.e., the number of statistically significant components, commonly referred to as the signal, as opposed to the noise; or (ii) a pair of successive components  $(k_0, k_0 + 1)$  for which  $\lambda_{k_0} \approx \lambda_{k_0 + 1}$ , which might capture, as we shall see, a possibly cyclic mode of behavior of the system (see Section 3). The whole set of RCs,  $\mathcal{K} = \{k : 1 \leq k \leq M\}$ , gives the complete reconstruction of the time series; it simply corresponds to the counterpart, for SSA, of Parseval's theorem for a Fourier series (or integral) or to wavelet reconstruction. The common, less mathematical notation for the reconstructed component  $r_k$  is RC  $k$ , and for a sum of several, consecutive RCs — from index  $k$  to index  $k'$  — it is RCs  $k$ – $k'$ .

From the viewpoint of signal processing, the RCs can be considered as filtered time series, where the filtering is *data adaptive*, being given by the projection onto the eigenvectors. Eq. (4) can be interpreted as a finite-impulse response (FIR) filter (Oppenheim and Schaffer, 1989), with  $\boldsymbol{\rho}_k$  being an FIR filter of length  $M$ . Next, the PCs  $A_k(t)$  obtained in Eq. (4) are time-reversed, cf. Eq. (5), and the FIR filter is run again through them. After this second filter pass, the correct chronological order is restored by reversing the filtered result  $r_k(t)$  once more. This procedure is called *forward-backward filtering*, and it is known to preserve the phase relations.

The filtering by projection onto each eigenvector  $\boldsymbol{\rho}_k$  is thus neutral in phase

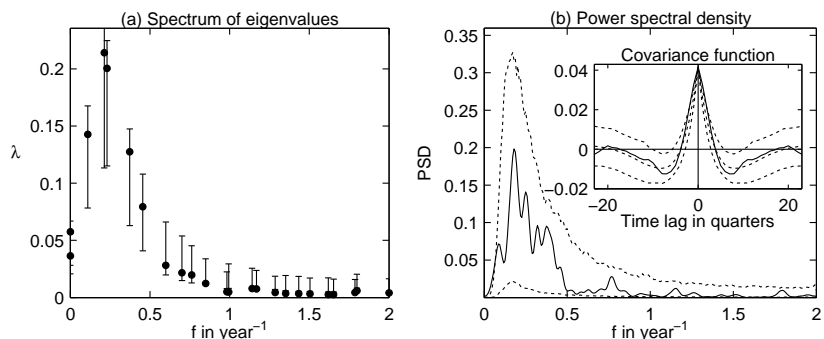


Figure 2: Univariate spectral analysis of U.S. gross domestic product (GDP). (a) Eigenvalue spectrum of  $\lambda_k$  (circles) vs. dominant frequency of the associated eigenvector  $\tilde{\rho}_k$ , with window width  $M = 24$  quarters; the error bars indicate the significance levels (cf. Sec. 3.1). (b) Power spectral density (PSD) estimate (solid lines) using Welch’s averaged periodogram method, with a Hamming window of length 128 quarters and 75% overlap (Priestley, 1991); the dashed lines indicate the significance levels. Inset: Covariance estimates and their significance levels; the solid line is the covariance of the GDP data, the dotted line is the corresponding AR(1) estimate. The upper and lower significance levels in both panels and in the inset are derived from the 2.5% and 97.5% percentiles of 1000 surrogate time series; see Sec. 3.1.

and acts only on the amplitudes. Hence, each RC  $r_k(t)$  and the original time series  $x(t)$  are in phase. In designing an appropriate band-pass filter, Baxter and King (1999) require, in particular, that this “filter should not introduce phase shifts.” Unlike their band-pass filter, with its data-independent weights given *a priori*, SSA is data adaptive, since the  $M$  filters are simply the eigenvectors of the auto-covariance matrix.

These eigenvectors extract a certain frequency band of the time series, and we thus assign to each pair  $(\lambda_k, \rho_k)$  a frequency, given by the maximum of the Fourier transform of  $\rho_k$ , cf. Vautard et al. (1992). Instead of plotting each eigenvalue vs. its rank, we can plot it vs. its dominant frequency. This approach provides a complementary perspective on SSA in terms of an analogy with classical spectral analysis; still, the eigenvector pairs associated with oscillatory modes are more flexible than the fixed pairs of sine and cosine functions that appear in Fourier analysis.

When analyzing the trend residuals of GDP alone, we observe a maximum in the spectrum of eigenvalues at the usually reported mean business cycle length of 5–6 years (Figure 2a). This result is in agreement with those of classical spectral density analysis (Figure 2b), which also exhibit a maximum around the same period, for various power spectral density (PSD) estimation algorithms that we have tested. At this point, though, the trend residuals are subject to the [Nelson and Kang \(1981\)](#) criticism of spurious cycles and so we have to perform additional tests before relying on the results; see [Sec. 3.1](#).

### 2.3. Multivariate SSA (M-SSA)

M-SSA extends univariate SSA to multivariate time series. [Broomhead and King \(1986b\)](#) proposed the use of an M-SSA version in the context of Mañé-Takens-style attractor reconstruction for nonlinear, deterministic dynamical systems. Once again, complete reconstruction is problematic, especially for large systems and in the presence of substantial amounts of noise. The more modest goal of reconstructing only the robust skeleton of the attractor led [Kimoto et al. \(1991\)](#), [Keppenne and Ghil \(1993\)](#), and [Plaut and Vautard \(1994\)](#) to formulate the algorithm described briefly herein; see also [Ghil et al. \(2002\)](#). In this section, we present the advantages of applying M-SSA to the data set at hand.

Instead of a single time series  $x(t)$ , we now observe multiple quantities simultaneously. Let  $\mathbf{x}(t) = \{x_d(t): d = 1 \dots D, t = 1 \dots N\}$  be now a vector time series of length  $N$ , with  $D$  channels. In the generalization of [\(2\)](#) we consider, beside all  $D$  auto-covariances  $\mathbf{C}_{d,d}$ , also all cross-covariances  $\mathbf{C}_{d,d'}$  to yield the *grand covariance matrix*  $\tilde{\mathbf{C}}$ :

$$\tilde{\mathbf{C}} = \begin{pmatrix} \mathbf{C}_{1,1} & \mathbf{C}_{1,2} & \dots & \mathbf{C}_{1,D} \\ \mathbf{C}_{2,1} & \mathbf{C}_{2,2} & \dots & \mathbf{C}_{2,D} \\ \vdots & \vdots & \mathbf{C}_{d,d'} & \vdots \\ \mathbf{C}_{D,1} & \mathbf{C}_{D,2} & \dots & \mathbf{C}_{D,D} \end{pmatrix}, \quad (7)$$

where  $\tilde{\mathbf{C}}$  is of size  $DM \times DM$  and the entries of the individual matrices  $\mathbf{C}_{d,d'}$

are given by

$$(c_{i,j})_{d,d'} = \frac{1}{\tilde{N}} \sum_{t=\max\{1,1+i-j\}}^{\min\{N,N+i-j\}} x_d(t)x_{d'}(t+i-j). \quad (8)$$

The denominator  $\tilde{N}$  depends on the range of summation, namely  $\tilde{N} = \min\{N, N+i-j\} - \max\{1, 1+i-j\} + 1$ .

As in the univariate case of Eq. (3) in Section 2.2, we now diagonalize the grand matrix  $\tilde{\mathbf{C}}$  to yield its eigenvalues  $\lambda_k$  and eigenvectors  $\tilde{\boldsymbol{\rho}}_k$ ,

$$\tilde{\mathbf{C}}\tilde{\boldsymbol{\rho}}_k = \lambda_k\tilde{\boldsymbol{\rho}}_k. \quad (9)$$

By solving this equation, we get  $DM$  pairs  $(\lambda_k, \tilde{\boldsymbol{\rho}}_k)$ , where each eigenvector  $\tilde{\boldsymbol{\rho}}_k$  of length  $DM$  is composed of  $D$  consecutive segments  $\boldsymbol{\rho}_k^d$ ,  $d = 1 \dots D$ , each of length  $M$ . These segments can be interpreted — like in the univariate case — as frequency-selective FIR filters.

The associated PCs are single-channel time series that are computed by projecting the multivariate time series  $(x_1, x_2, \dots, x_D)$  onto  $\boldsymbol{\rho}_k^d$ ,

$$A_k(t) = \sum_{d=1}^D \sum_{j=1}^M x_d(t+j-1)\rho_k^d(j), \quad k = 1 \dots DM. \quad (10)$$

In addition to the summation  $j$  over time as in the univariate SSA of Eq. (4), we have here a second summation with respect to the distinct channels  $d$ , representing a PCA. Hence, M-SSA combines two useful approaches: it extracts (1) major spectral components by means of SSA; and (2) major directions in phase space by means of PCA. In particular, setting  $M = 1$  reduces M-SSA to PCA.

As in the univariate case of Eq. (5), one can reconstruct, cf. [Plaut and Vautard \(1994\)](#), that part of each time series  $x_d(t)$  that is associated with a particular eigenvector  $\boldsymbol{\rho}_k^d$ , by

$$r_k^d(t) = \frac{1}{M_t} \sum_{j=L_t}^{U_t} A_k(t-j+1)\rho_k^d(j), \quad k = 1 \dots DM, d = 1 \dots D. \quad (11)$$

Like in single-channel SSA, the M-SSA result of Eq. (11) provides us with a set of  $DM$  RCs, but it does so for each of the  $D$  time series  $x_d(t)$ . Depending on the

information contained in the cross-covariances  $\mathbf{C}_{d,d'}$ , the RCs of different channels may or may not be correlated. In this way, M-SSA helps extract common spectral components from the multivariate data set, along with comovements of the channels. It is especially the inclusion of temporal correlations that makes M-SSA superior to PCA in the extraction of dynamical behavior, as we shall see forthwith.

With this brief introduction to M-SSA, we compare in Figure 1 the pre-processed time series (Figure 1a) with the M-SSA reconstruction (Figure 1b) and the PCA reconstruction (Figure 1c). Both the M-SSA and PCA reconstructions capture 75% of the total variance and extract coherent behavior manifest in the nine economic variables.

In contrast to PCA, the M-SSA results are much smoother, having removed small, non-essential fluctuations. In Sec. 4, we will further analyze the significance of the remaining, data-adaptively smoothed fluctuations for business cycle dynamics. Before doing so, we address now the critical issue of whether the longer-scale behavior in Figure 1b may and should be interpreted as a set of coherent oscillations. In this perspective, we pursue in the following section the statistical significance of oscillatory behavior in the single- and multi-channel SSA analysis of the U.S. aggregate indicators.

### 3. Oscillatory behavior and its statistical significance

The trend residuals in Figure 1 exhibit obviously more structure than pure white noise; we need, therefore, a stringent test to decide whether the visually apparent cyclical behavior can be attributed to random fluctuations or to a more regular oscillatory behavior, of possibly intrinsic origin. Cogley and Nason (1995), among others, have discussed in detail the effect of detrending and the generation of spurious cycles; their discussion was placed in the context of the detrending effect on a standard real business cycle (RBC) model, with no oscillatory dynamics. These authors showed, in particular, that the PSD of an HP-filtered random walk has a peak at a period of 7.6 years.

We follow [Cogley and Nason \(1995\)](#) and test against an autoregressive process of order one, denoted by AR(1), in order to verify the statistical significance of oscillations; this test is a well-established, standard tool in the geosciences, too, cf. [Ghil et al. \(2002\)](#). Such AR(1) processes exhibit greater variance at low frequencies, with a maximum at zero frequency and no other, local maxima, while detrending with the HP filter — even when applied to the simulations of an RBC model with no oscillatory dynamics — yields a maximum in the PSD around the commonly reported business cycle length of five years.

We thus fit an AR(1) process to the detrended and standardized BEA data set and pass the output through an HP filter, in such a way as to preserve certain stylized facts, including the auto- and cross-correlations of the data. By means of Monte Carlo simulation, we test whether oscillatory modes exist that cannot be explained by the fluctuations of the surrogate time series. [Allen and Smith \(1996\)](#) introduced and discussed in detail this method, referred to as *Monte Carlo SSA*.

In the univariate case, we compare the Monte Carlo SSA results with those of standard spectral density estimation to verify further that the two approaches are consistent (Sec. 3.1). Next, we present the advantages of a full, multivariate SSA in finding robust oscillatory modes (Sec. 3.2), whose characteristics are less obvious in a univariate analysis.

### 3.1. Univariate time series: the GDP

We consider first the simple case of a univariate time series, in which we do not have to take comovements into account. The analysis focuses on GDP, usually considered to be the most important macroeconomic indicator.

First off, we have to find an appropriate null hypothesis of a stochastic process that captures the typical “stylized facts” of our original data set, but in which we know that no oscillations are present. The natural choice is to fit an AR(1) process to the time series,

$$X(t) = a X(t - 1) + \sigma_0 \epsilon(t), \tag{12}$$

with  $\epsilon(t)$  being Gaussian white noise of variance  $\sigma = 1$ . In the case of GDP residuals, we estimate the regression coefficient to be  $a = 0.82$ , while the estimated variance is  $\sigma_0 = 0.04$ .

In this estimation procedure, we have to consider the influence of the HP filter that we apply to the AR(1) output. In doing so, we choose  $a$  such that the mean-square distance between the covariance function of the GDP (solid line in the inset of Figure 2b) and the covariance function of the HP filtered AR(1) output (dotted line) is minimal. Given the model parameter  $a$ , we estimate  $\sigma_0$  with the method proposed by Allen and Smith (1996), and determine 1000 realizations of Eq. (12), with the same length of 52 years as the data set. Finally, we filter all realizations with exactly the same HP filter as the actual GDP data and standardize the filtering result in order to obtain our set of surrogate time series.

In Figure 2b, we have estimated the PSD of the GDP residuals displayed in Figure 1a. The PSD estimate (solid line) is clearly much higher around a five-year period. On the other hand, we have also estimated the PSD for each of the surrogate time series and derived significance levels for each frequency from the 2.5% and 97.5% percentiles (dashed lines). These significance levels also show high power around five years, and the PSD estimate falls entirely between them.

We have applied several PSD estimation methods and conclude that, in this univariate analysis, the GDP residuals cannot be distinguished from the null hypothesis of an HP-filtered AR(1) process, and the high PSD values around five years could be due to the detrending of an otherwise stable model with exogenous excitation. Such a detrending effect is in complete agreement with the findings of Cogley and Nason (1995) and Nelson and Kang (1981).

The same lack of statistical significance holds — as expected from the Wiener-Khinchin theorem that links the PSD and the lag-covariance function of a time series — for the latter: the swing below zero for the surrogate time series is likewise due to the HP filter’s effect; see again the small inset in Figure 2b. Note, indeed, that the auto-correlation function of a pure AR(1) process

is proportional to  $1/\tau^2$ , with  $\tau$  being the time lag, and thus only take positive values (Blackman and Tukey, 1958). The preliminary conclusion is that, for the univariate GDP time series at hand, we cannot falsify the null hypothesis of an AR(1) process.

This negative finding can also be confirmed by Monte Carlo SSA. This method tests whether an eigenvalue  $\lambda_k$  captures more partial variance in the direction of the corresponding eigenvector  $\rho_k$  than present in the null hypothesis (Allen and Smith, 1996). To derive the significance level in the eigenvalue spectrum, we project the covariance matrix  $\mathbf{C}_S$ , estimated for each surrogate time series  $\mathbf{x}_S(t)$  as in Eq. (2), onto the eigenvectors  $\rho_k$  of the original time series

$$\mathbf{\Lambda}_S = \mathbf{E}^\top \mathbf{C}_S \mathbf{E}, \quad (13)$$

where the  $\rho_k$  are the columns of  $\mathbf{E}$ , and  $(\cdot)^\top$  denotes the transpose of a vector or a matrix.

Since Eq. (13) is not the eigendecomposition of the particular realization  $\mathbf{C}_S$ , the matrix  $\mathbf{\Lambda}_S$  is not necessarily diagonal, as it would be for  $\mathbf{C}$ . Instead,  $\mathbf{\Lambda}_S$  provides a measure of the discrepancy between the surrogate time series  $\mathbf{x}_S(t)$  and the original time series  $\mathbf{x}(t)$ . By computing the percentiles of the diagonal-element distribution from a set of  $\mathbf{\Lambda}_S$ , we derive significance levels for each eigenvalue  $\lambda_k$ . In Figure 2a, these levels of significance are indicated as vertical bars and we see, as in the case of the PSD, that all the eigenvalues  $\lambda_k$  fall within these error bars. This is not surprising, given the already stated analogy between SSA and PSD, although SSA is more flexible, due to the data-adaptive eigenvectors it uses as a basis. In the following subsection, we will demonstrate nonetheless that, by including additional information from other macroeconomic indicators, it is still possible to reject a reasonable multivariate null hypothesis.

### 3.2. Multivariate time series

In the multivariate case, the hypothesis that we test against has to be modified. Allen and Robertson (1996) have proposed to fit an independent AR(1)

Table 1: Null-hypothesis parameters

Variable	AR(1) parameters		Time lag behind GDP
	$a_d$	$\sigma_d$	$\Delta_d$ (quarters)
GDP	0.82	0.040	0
Investment	0.92	0.028	1
Employment	0.96	0.021	0
Consumption	0.88	0.034	0
Total wage	0.95	0.024	0
$\Delta$ (Inventories)	0.61	0.055	0
Price	0.99	0.012	7
Exports	0.95	0.023	1
Imports	0.83	0.039	1

process to each separate time series. A simple inspection of the trend residuals in Figure 1a, though, suggests comovements that should be taken into account in formulating the null hypothesis.

An alternative would be to use a vector AR(1) model. When testing for significant oscillations, we need, however, a null-hypothesis model that does not support oscillations. Vector AR models, though, may support oscillations even for order one; when present, these are referred to as principal oscillation patterns (von Storch et al., 1995; Penland and Matrosova, 2001). We keep, therefore, the idea of fitting individual AR(1) processes, but build characteristics of the auto- as well as the cross-correlations of the given time series into each one of the fits.

We thus start by fitting a scalar AR(1) process to each of the time series, as in the univariate case:

$$X_d(t) = a_d X_d(t-1) + \sigma_d \epsilon_d(t). \quad (14)$$

The estimated parameters for each macroeconomic indicator are listed in Table 1, and the cross-covariances between the indicators are plotted in Figure 3.

Next, we include information on these cross-covariances into the null hypothesis by coupling the noise residuals  $\epsilon_d(t)$  of the individual AR(1) processes

$X_d(t)$  in Eq. (14). We do so by allowing for temporal lags  $\Delta_d$  between the GDP, denoted by  $x_1(t)$ , and the other variables,  $x_{d \neq 1}(t)$ , and set each lag  $\Delta_d$  to equal the one at which the cross-covariance function between  $x_1(t)$  and  $x_d(t)$  reaches its maximum. Such a lag is especially necessary for the price, for which the maximal cross-covariance with GDP occurs at about seven quarters (cf. Figure 3).

The covariance matrix  $\mathbf{R}$  that we take into account for the coupling of the innovation processes  $\epsilon_d(t)$  has elements  $\mathbf{R}_{d,d'}$  given by

$$\mathbf{R}_{d,d'} = \frac{1}{\tilde{N}_{d,d'}} \sum_{t=\max\{1,1+\Delta_{d'}-\Delta_d\}}^{\min\{N,N+\Delta_{d'}-\Delta_d\}} x_d(t)x_{d'}(t+\Delta_{d'}-\Delta_d); \quad (15)$$

the denominator  $\tilde{N}_{d,d'}$  here depends on the range of summation, namely  $\tilde{N}_{d,d'} = \min\{N, N + \Delta_{d'} - \Delta_d\} - \max\{1, 1 + \Delta_{d'} - \Delta_d\} + 1$ , cf. Eq. (8). Cholesky decomposition yields  $\mathbf{R} = \mathbf{L}^\top \mathbf{L}$  and we derive correlated innovation processes from

$$(\epsilon_1(t), \dots, \epsilon_d(t + \Delta_d), \dots, \epsilon_D(t + \Delta_D))^\top = \mathbf{L}^\top (\xi_1(t), \dots, \xi_d(t), \dots, \xi_D(t))^\top, \quad (16)$$

with the  $\xi_d$  being independent white-noise processes. We thus build certain time-lag patterns found in the given data set into the set of  $D = 9$  individual AR(1) processes. Finally, we pass the time series so generated through the HP filter to remove low frequencies, normalize it to the same standard deviation as the data set, and thus obtain the surrogate time series we test against.

Figure 3 compares the auto- and cross-covariance functions with respect to GDP of the original and the surrogate time series. We observe that the lead-lag relations among economic indicators that are associated with typical stylized facts of business cycles (Zarnowitz, 1985; Hallegatte et al., 2008) are reproduced by the null-hypothesis model, and that the covariance functions of the data set lie almost, but not quite, within the fluctuations of the multivariate AR(1) model. As in the univariate case, the HP filter introduces a swing below zero, whose minimum is at approximately five quarters and which would lead to spurious cycles with a length of roughly 20 quarters.

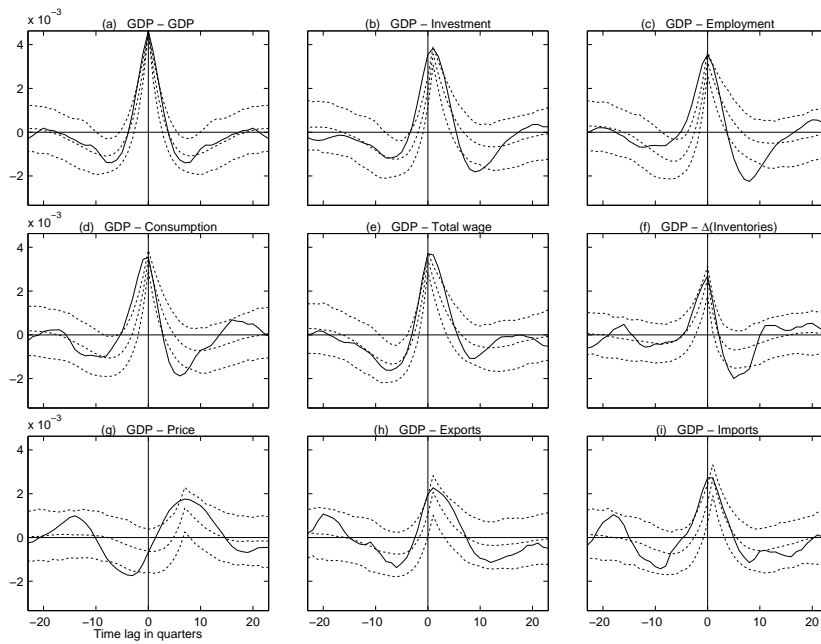


Figure 3: Auto- and cross-covariance functions of the nine U.S. economic indicators with respect to GDP (solid lines). The dashed lines are the significance levels (2.5% and 97.5%), and the dotted line is the median from the realization of 1000 surrogate time series; see text for details.

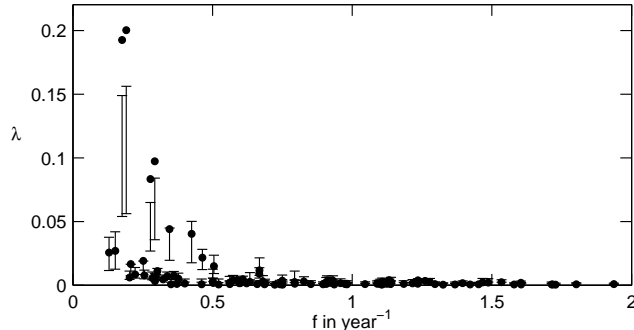


Figure 4: Spectrum of M-SSA eigenvalues (filled circles) using all nine U.S. indicators, with  $M = 24$ . The error bars indicate the significance levels, derived from the 2.5% and 97.5% percentiles of 1000 multivariate surrogate time series.

To derive significance levels for the M-SSA eigenvalues, we determine again — from Eq. (7) and for each surrogate time series — the grand covariance matrix  $\tilde{\mathbf{C}}_S$  and project it onto the eigenvectors of the original time series,  $\tilde{\mathbf{\Lambda}}_S = \tilde{\mathbf{E}}^\top \tilde{\mathbf{C}}_S \tilde{\mathbf{E}}$ , as in Eq. (13). From the distribution of the elements on the main diagonal of  $\tilde{\mathbf{\Lambda}}_S$ , we derive as before the significance levels for each eigenvalue  $\{\lambda_k : 1, \dots, DM\}$ . These, along with the actual eigenvalues associated with the original data set, are plotted in Figure 4.

As in the case of GDP in Figure 2a, we observe in Figure 4 higher levels of surrogate eigenvalues near a five-year period. But this time the oscillatory five-year mode in the data set clearly exceeds the significance level of the null hypothesis and can no longer be explained by spurious cycles that would be induced by inappropriate detrending.

We have also tested the robustness of the present results by using different values of the window width,  $M = 20, 30, 40$  and  $50$ : it turns out that the leading pair of eigenvalues always describes a significant oscillatory mode at a period of about five years (not shown). On the other hand, the three-year oscillation that is reflected in the second pair of eigenvalues in Figure 4 is less robust.

We have performed additional, exhaustive tests — as proposed by [Allen and Smith \(1996\)](#) — to cope with the problem of overestimating large eigenvalues in

SSA and have found further evidence that the five-year oscillatory pair is indeed statistically significant at the 95% level. We focus, therefore, in the next section on this oscillatory mode and investigate its role in business cycle dynamics.

#### 4. Changes in business cycle dynamics

The existence of an oscillatory pair indicates that the trajectory of the system that produced the time series is attracted by a limit cycle in phase space; in this case, the cycle has a period of about five years. Although this limit cycle does not explain the full dynamical behavior — e.g., the first two eigenvalues capture only 40% of the total variance — RCs 1-2 give a good approximation of the dynamical behavior of GDP, for example (Figure 5a).

In order to better understand the role of the five-year oscillatory mode in the processes of expansion and recession, we study the evolution of its variance throughout the cycle. [Plaut and Vautard \(1994\)](#) introduced the concept of *local variance fraction*  $V_{\mathcal{K}}(t)$ ,

$$V_{\mathcal{K}}(t) = \frac{\sum_{k \in \mathcal{K}} A_k(t)^2}{DM \sum_{k=1} A_k(t)^2}, \quad (17)$$

which quantifies how much of the total variance is captured by the set  $\mathcal{K}$  of orthogonal PCs at a given time  $t$ .

This index is especially useful in measuring the relative amplitude of an oscillatory activity, described here by PCs 1-2, within a time window of width  $M$ . The PCs are used in Eq. (17), rather than the RCs, since it is the former that give us the projection of the multivariate time series onto the eigenvector pairs. As already noted in connection with Eq. (4), we consider the PCs as centered, i.e. starting at  $M/2$ .

Figure 5 shows this index, along with the NBER-defined recessions, for PCs 1-2 in panel (b) and for PCs 3–150 in panel (c). The sum of these two, i.e. of PCs 1–150, is shown in panel (b) as the dash-dotted line and it captures 99% of the total variance. Starting after 1970, it is quite remarkable that the

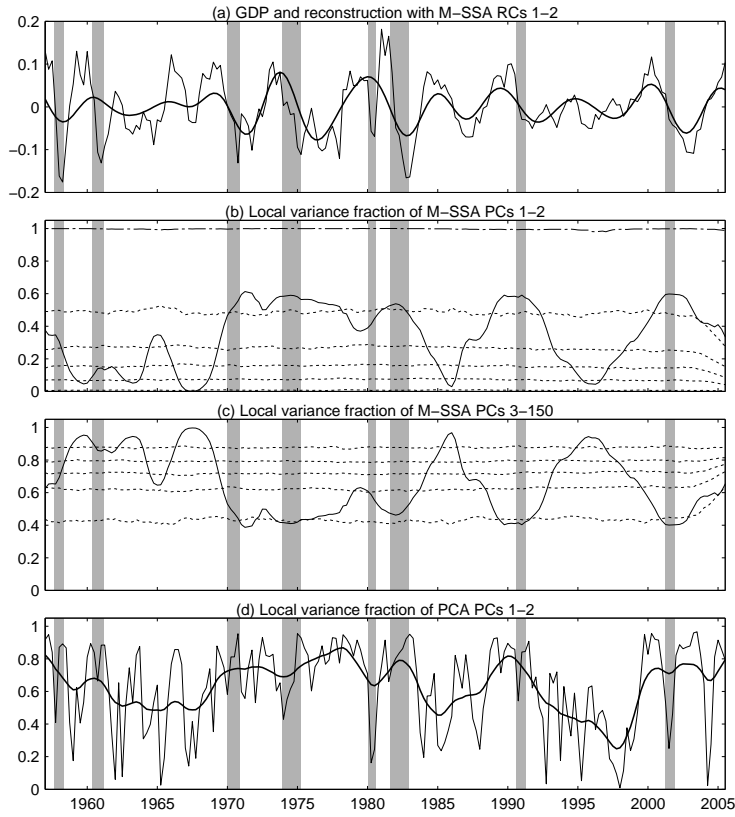


Figure 5: (a) Pre-processed GDP data set (light solid line) and its reconstruction with RCs 1-2 of our M-SSA analysis (heavy solid line). (b,c,d) Local variance fraction  $V(t)$ : (b) for M-SSA PCs 1-2 (solid line) and PCs 1-150 (near-total variance, dash-dotted line); (c) for M-SSA PCs 3-150 (solid line); and (d) for PCs 1-2 of a PCA analysis (light solid line), as well as after smoothing with a two-year moving average (heavy solid line). The dashed lines in panels (b) and (c) give the 2.5%, 25%, 50%, 75%, and 97.5% percentiles based on 1000 surrogate time series. The shaded vertical bars indicate NBER-defined recessions.

fraction of the five-year oscillatory mode in PCs 1-2 is high during recessions and low during expansions. This fraction  $V_{\mathcal{K}=\{1,2\}}(t)$  shows that during the recessions, the trajectory of the U.S. economy described by the BEA data set stays closer to a suspected five-year limit cycle — like the one in the Non-Equilibrium Dynamic Model (NEDyM) of Hallegatte and Ghil (2008) or in other endogenous business cycle models (Chiarella et al., 2005) — while this trajectory reveals more complex behavior during expansions.

Furthermore, Figure 5 suggests a change in the system’s dynamics in the 1980s. During the 1970s, PCs 1-2 capture roughly 50% of the variance or more over the full decade, while from 1980 on, PCs 1-2 play a significant role only during recessions. This change falls into the same time interval as the “great moderation,” during which volatility in GDP growth diminished markedly (Kim and Nelson, 1999; McConnell and Perez-Quiros, 2000; Stock and Watson, 2002; Kim et al., 2004).

There has been considerable debate on the cause of this shift, as well as on the expected duration of the U.S. economy’s new mode of functioning; in particular it has been proposed that this moderate behavior terminated in 2007, i.e. before and during the “great recession” of 2008-2009. In any case, our results are at least consistent with the hypothesis of structural changes in the 1980s, and our M-SSA methodology can help provide sophisticated analysis tools to determine whether and when the great moderation ended, once additional BEA data become available.

In order to assess whether the variability of the local variance fraction  $V_{\mathcal{K}}(t)$  can be explained by the random fluctuations of the null hypothesis, we examine the significance of this variability by the same type of procedure as for the eigenvalues in Eq. (13). To wit, we project each surrogate set  $\mathcal{K}$  of PCs — e.g., PCs 1-2, 3–150 and 1–150 — onto the data eigenvectors  $\tilde{\rho}$ , in the same way as for the data set in Eq. (10). The resulting time series are, once more, not orthogonal, and their covariance matrix  $\tilde{\mathbf{A}}_{\mathcal{S}}$  is not diagonal, but it allows us — as in the case of the significance levels for the eigenvalues — to assess the distance from the data PCs.

We calculate, for each set of surrogate time series  $\mathcal{K}$ , the local variance fraction  $V_{\mathcal{K}}(t)$ , in the same way as for the data set in Eq. (17), and derive for each epoch  $t$  the significance levels from the percentiles (Figs. 5b,c, dashed lines). Since the AR(1) processes are stationary, these levels are supposed to be constant; this stationarity is seen in fact in Figure 5, except near the end of the time series, i.e. starting at  $t \simeq N - M$ , where  $M = 24$  quarters.

In contrast to the approximate constancy of  $V(t)$  for the AR(1) processes, the five-year oscillatory mode in the BEA data exhibits much greater variance during the recessions, when it does exceed the 97.5% significance level. The variance in PCs 3–150 is also larger than can be explained by the null hypothesis, with  $V(t)$  values that are significantly larger than the 97.5% percentile during expansions and smaller than the 2.5% percentile during recessions, respectively.

We have further examined the variability of  $V(t)$  during the whole 1954–2005 interval by using other quantities, such as standard deviation and interquartile range (not shown here). All these estimates confirm that the U.S. macroeconomic indicators exhibit larger variability than can be explained by the random fluctuations of our null hypothesis.

A similar phenomenon can also be identified by applying PCA to the data (Figure 5d). Although, at first glance, the local variance fraction of the leading two PCs of PCA fluctuates wildly, with no apparent link to the business cycle (light solid line), smoothing with a two-year moving-average filter (heavy solid line) does indeed produce a behavior comparable to that in Figure 5b. It would, however, been difficult to guess that from the unsmoothed results, and the moving-average filtering was only inspired by the M-SSA results in panels (b) and (c), which did not require any additional post-processing.

## 5. Concluding remarks

In this article, we proposed a novel methodology — namely singular spectrum analysis (SSA) and its multivariate extension (M-SSA) — to study business cycles in a consistent and multivariate way, which allowed us to reconcile

and combine the NBER definition of recessions with quantitative analysis. We applied this methodology to nine U.S. macroeconomic indicators available from the BEA for 52 years (1954–2005); see Section 2.1 and Figure 1 there.

This analysis leads to three major conclusions that concern, respectively: (i) the presence of genuine periodicity in macroeconomic behavior and its deterministic causes; (ii) the essential role of comovements of economic aggregates in the proper definition of business cycles; and (iii) the dependence of economic “volatility” on the phase of the business cycle. We describe these conclusions in greater detail below.

*Genuine periodicity and its deterministic causes.* In their work about the “real facts” and monetary myths of business cycles, [Kydland and Prescott \(1998\)](#) discussed the origin of business cycles in terms of the [Slutzky \(1937\)](#) theory of random shocks. In the simplest RBC models, cyclicity originates exclusively from productivity shocks that can be modeled by a simple random walk. [Cogley and Nason \(1995\)](#) have, moreover, argued that the spurious appearance of business cycle dynamics can be generated by the HP filter even if none is present, even in a random walk.

Indeed, in agreement with the findings of [Cogley and Nason \(1995\)](#), a simple univariate analysis of GDP does not reveal any significant oscillatory modes (see Figure 2). The multivariate, M-SSA version of our analysis, however, permits the systematic, self-consistent use of a larger amount of information about macroeconomic behavior; it allows us, therewith, to identify a five-year oscillatory mode with high statistical confidence (see Figure 4). This mode cannot be explained by artificial effects due to detrending by the HP filter, and a random-walk-driven model of business cycles has to be questioned in the light of the results obtained in the present investigation.

It is well known, as already mentioned at the beginning of Section 3.2, that vector AR(1) processes can possess oscillatory solutions, due to the presence of pairs of complex conjugate eigenvalues  $(\lambda_k, \lambda_{k+1}) = (\lambda_k^{(r)} \pm \lambda_k^{(i)})$  in the spectrum

of the matrix  $\mathbf{A} = (a_{ij})$  that characterizes such a process,

$$\mathbf{X}(t) = \mathbf{A} \mathbf{X}(t-1) + \mathbf{\Sigma} \epsilon(t); \quad (18)$$

here  $\mathbf{\Sigma}$  is a covariance matrix multiplying the noise vector  $\epsilon$ . For a stationary AR(1) process, all the real parts  $\lambda_k^{(r)}$  of the eigenvalues of  $\mathbf{A}$  must be negative, and the damped oscillations are maintained at a statistically constant amplitude by the noise  $\epsilon$ .

In practice, such a stochastically driven oscillator might be hard to distinguish from a purely deterministic, possibly chaotic one. But the term  $\mathbf{A} \mathbf{X}(t-1)$  in the former case, on the right-hand side of Eq. (18), still captures the manifestation of a coupled pair of deterministic feedbacks, one positive, the other negative, whether linear or nonlinear, noise-driven or not, as the ultimate cause of any complex conjugate pair of eigenvalues  $(\lambda_k^{(r)} \pm \lambda_k^{(i)})$ .

A major result of our M-SSA study thus points rather unambiguously to the presence of such deterministic effects in the business cycles of the U.S. economy. We conclude, therefore, that business cycles cannot be explained by exogenous shocks alone and arise from complex interactions between endogenous dynamics and exogenous perturbations.

*Comovements of macroeconomic aggregates.* The role of the additional information provided by the M-SSA analysis emphasizes the need to understand business cycles as a phenomenon that is not limited to GDP variations, but involves all aspects of the economy; it is reflected, therefore, in the comovements of several macroeconomic aggregates. In the present study, we have performed an innovative, quantitative analysis of the BEA data set that is consistent with the NBER definition of the business cycle, inasmuch as it is entirely multivariate and takes into account the lead-and-lag relationships between the various indicators present in the data (see Table 1 and Figure 3).

*State-dependent fluctuations.* M-SSA also allowed us to provide further insight into the underlying macroeconomic dynamics, and especially into the crucial question of the complex interplay between endogenous dynamics and exogenous shocks. We showed that the U.S. economy changes its behavior from

one phase of the business cycle to another: the recession phase is dominated by the five-year mode, while the expansion phase exhibits more complex dynamics, with higher-frequency modes coming into play (see Figure 5). This type of behavior cannot be explained by the random fluctuations that drive a simple stationary RBC model, in the absence of endogenous oscillatory dynamics.

Pursuing further the implications of our results, let us assume that the dynamics of the U.S. economy can indeed be decomposed into a five-year cycle and more complex, higher-frequency behavior superimposed on this cycle: the latter suggests a higher volatility of the business cycle during expansions. This inference is consistent with intuition, since recessions are phases of underutilized production capacities; hence economic variables, such as GDP and employment, are likely to be less sensitive to exogenous shocks, be it natural disaster (Hallgatte and Ghil, 2008), monetary or fiscal policy shocks, or productivity shocks. During expansions, on the other hand, production is closer to saturation, and exogenous shocks — whether positive or negative — are likely to have a bigger impact. This “vulnerability paradox” was also highlighted by Ghil et al. (2011).

This variable-volatility pattern is, in fact, not at variance with the findings of French and Sichel (1993). These authors have modeled the variance of the residuals on a long-term trend, without decomposing these residuals into cyclical and non-cyclical behavior, as we do here, and found higher variance during epochs of recession. In the present paper, we study the fluctuations superimposed on the sum of the long-term trend, plus a possible cyclical component. It is the variance of the fluctuations so defined that is largest during expansions.

The next step in our research program is to investigate whether the change in the economy’s dynamical behavior between boom and bust also leads to different types of response to exogenous shocks. This question is fundamental in attempting to evaluate the efficiency of economic policy in different phases of the business cycle.

### *Acknowledgments*

It is a pleasure to thank J.-C. Hourcade for informative and stimulating discussions and J. H. Stock for detailed and constructive comments on an earlier version of the paper. This work was supported by a CNRS post-doctoral fellowship to AG, the Groupement d'Intérêt Scientifique (GIS) Réseau de Recherche sur le Développement Soutenable (R2DS) of the Région Ile-de-France, and the Chaire Développement Durable of the Ecole Polytechnique.

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