

**Have approx. analytic solution for convective regions:**  
convective constrains  $T \Rightarrow$  vertical structure of baroclinic  
pressure gradients

$\Rightarrow$  vertical structure of  $v \Rightarrow$  vertical structure of  $\omega$

Extend to full nonlinearity, non-convective regions,...

Use **analytic solutions** for leading **basis functions** in  
Galerkin expansion in vertical

$$T = T_r(p) + \sum_{k=1}^K a_k(p) T_k(x, y, t) + T_R,$$

• Horizontal gradients of  $T$  matter; specify reference state  $T_r(p)$  to improve accuracy.

• Simplest case: 1 basis function in  $T, q$

Extra basis function for external mode  $\Rightarrow$  2 in  $v$

# Model Summary – QTCM equations

$$\partial_t \mathbf{v}_1 + D_{V1}(\mathbf{v}_0, \mathbf{v}_1) + f\mathbf{k} \times \mathbf{v}_1 = -\kappa \nabla T_1 - \text{stress}$$

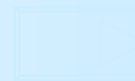
$$\partial_t \mathbf{v}_0 = \dots \text{ (barotropic component)}$$

$$\hat{a}_1(\partial_t + D_{T1})T_1 + M_{S1} \nabla \cdot \mathbf{v}_1 = \langle Q_c \rangle + \text{Rad} + H$$

$$\hat{b}_1(\partial_t + D_{q1})q_1 + M_{q1} \nabla \cdot \mathbf{v}_1 = \langle Q_q \rangle + E$$

Moisture sink and convective heating

$$-\langle Q_q \rangle = \langle Q_c \rangle = \varepsilon_c(q_1 - T_1)$$



# Quasi-equilibrium schemes

Posit that bulk effects of convection tend to establish statistical equilibrium among buoyancy-related fields

Approach here depends on **convection** tending to **constrain vertical structure of temperature** field.

For now: **Smoothly posed convective adjustment**

Convective heating: (Betts 1986; Betts & Miller 1986)

$$Q_c = (T_c - T) / \tau_c$$

$\tau_c$  time scale of convective adjustment

$T_c$  convective reference profile; **depends on  $h_b$**  for convection arising out of PBL

$h_b$  planetary boundary layer (PBL) moist static energy after adjustment by downdrafts to satisfy energy constraint

$T_c$  typically moist adiabat or closely related

Can be expanded about a reference state,  $T(p)$

$$T_c = T_c + A(p)h_b' + \text{higher order}$$

$A(p)$  vertical dependence of the moist adiabat perturbation per  $h_b$  perturbation

Tends to reduce CAPE (convective available potential energy)

# Analytical solution under quasi-equilibrium convective constraints

If  $T$  constrained to be close to QE temp  $T^c$

$$T \approx T^c \approx \underline{T_r^c} + A(p)T_1^c$$

Primitive equations, momentum + hydrostatic:

$$\begin{aligned} (\partial_t + D_m)v + f\mathbf{k} \times v &= -\kappa \nabla \int_p^{p_o} T d \ln p + \nabla \phi_o \\ &\approx -\kappa \int_p^{p_o} \underline{A(p)} d \ln p \underline{\nabla T_1^c} + \nabla \phi_o \end{aligned}$$

baroclinic pressure gradients have strongly constrained vertical structure

# Analytic solution in deep convective regions (cont.)

Vertical structure of baroclinic pressure gradients  
 $\Rightarrow$  structure of baroclinic wind  $V_1$ . With barotropic component  $\Rightarrow$

$$v = v_o(x,y,p,t) + V_1(p)v_1(x,y,t)$$

Continuity eqn.  $\Rightarrow$   $\omega = \Omega_1(p) \nabla \cdot v_1$

The moist static energy eqn. becomes

$$(\partial_t + \mathbf{D})(\hat{T} + \hat{q}) + M \nabla \cdot v_1 = F_{net}$$

where  $M$  is the gross moist stability  $M = \langle \Omega \partial_p h \rangle$

NB: Have not yet used convective closure on moisture.