Have approx. analytic solution for convective regions:

convective contrains $T \Rightarrow$ vertical structure of baroclinic pressure gradients

$\Rightarrow$ vertical structure of $v \Rightarrow$ vertical structure of ω

Extend to full nonlinearity, non-convective regions,…

Use analytic solutions for leading basis functions in Galerkin expansion in vertical

$$T = T_r(p) + \sum_{k=1}^{K} a_k(p) T_k(x, y, t) + T_R,$$

- **Horizontal gradients of T matter; specify reference state $T_r(p)$ to improve accuracy.**
- **Simplest case: 1 basis function in T, q**

Extra basis function for external mode $\Rightarrow$ 2 in v

Model Summary – QTCM equations

$$\partial_t \boldsymbol{v}_1 + D_{V1}(\boldsymbol{v}_0, \boldsymbol{v}_1) + f\boldsymbol{k} \times \boldsymbol{v}_1 = -\kappa \nabla T_1 - \text{stress}$$

$$\partial_t \boldsymbol{v}_0 = \ldots \textit{ (barotropic component)}$$

$$\hat{a}_1(\partial_t + D_{T1})T_1 + M_{S1}\nabla \cdot \boldsymbol{v}_1 = \langle Q_c \rangle + \text{Rad} + H$$

$$\hat{b}_1(\partial_t + D_{q1})q_1 + M_{q1}\nabla \cdot \boldsymbol{v}_1 = \langle Q_q \rangle + E$$

Moisture sink and convective heating

$$-\langle Q_q \rangle = \langle Q_c \rangle = \varepsilon_c (q_1 - T_1)$$

Quasi-equilibrium schemes

Posit that bulk effects of convection tend to establish statistical equilibrium among buoyancy-related fields

Approach here depends on **convection** tending to **constrain vertical structure of temperature** field.

For now: **Smoothly posed convective adjustment**

Convective heating: (Betts 1986; Betts & Miller 1986)

$$Q_c = (T_c - T)/\tau_c$$

τ_c time scale of convective adjustment

T_c convective reference profile; depends on h_b for convection arising out of PBL

h_b planetary boundary layer (PBL) moist static energy after adjustment by downdrafts to satisfy energy constraint

T_c typically moist adiabat or closely related

Can be expanded about a reference state, $T(p)$

$$T_c = T_c + A(p)h_b' + \textbf{higher order}$$

$A(p)$ vertical dependence of the moist adiabat perturbation per h_b perturbation

Tends to reduce CAPE (convective available potential energy)

Analytical solution under quasi-equilibrium convective constraints

If T constrained to be close to QE temp T^c

$$T \approx T^c \approx \underline{T_r^c + A(p)T_1^c}$$

Primitive equations, momentum + hydrostatic:

$$(\partial_t + D_m)v + f\mathrm{k}\times v = -\kappa\nabla\int_p^{p_o} Td\ln p + \nabla\phi_o$$

$$\approx -\kappa\int_p^{p_o} \underline{A(p)}d\ln p\nabla \underline{T_1^c} + \nabla\phi_o$$

baroclinic pressure gradients have strongly constrained vertical structure

Analytic solution in deep convective regions (cont.)

Vertical structure of baroclinic pressure gradients

$\Rightarrow$ structure of baroclinic wind V_1. With barotropic component $\Rightarrow$ $v = v_o(x,y,p,t) + V_1(p)v_1(x,y,t)$

Continuity eqn. $\Rightarrow$ $\omega = \Omega_1(p)\, \nabla\cdot v_1$

The moist static energy eqn. becomes

$$(\partial_t + \mathbf{D})(\hat{T} + \hat{q}) + M\nabla\cdot v_1 = F_{net}$$

where M is the gross moist stability $M = \langle \Omega \partial_p h \rangle$

NB: Have not yet used convective closure on moisture.