Controlled precipitation of radiation belt electrons

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[1] First-order estimates indicate that the lifetime of energetic (a few MeV) electrons in the inner radiation belts (e.g., near $L = 2$) may be significantly reduced by in situ injection of whistler mode waves at radiated power levels of a few kW at frequencies of a few kHz. Our estimates are based on previously published results concerning the effect on the electron lifetimes of VLF signals from ground-based VLF transmitters operating in the 17–23 kHz range. Waves at lower frequencies (a few kHz) can drive diffusion rates that are higher by a factor of as much as ~30 and can also be efficiently stored in the magnetospheric cavity, resulting in additional effective enhancement of wave power density of a factor of ~16. This wave power enhancement is also expected to enhance the MeV electron diffusion rates. *INDEX TERMS:* 2716 Magnetospheric Physics: Energetic particles, precipitating; 2483 Ionosphere: Wave/particle interactions; 6984 Radio Science: Waves in plasma; 2730 Magnetospheric Physics: Magnetosphere—inner; *KEYWORDS:* electron precipitation, radiation belts, VLF transmitters, wave-particle interactions, pitch angle scattering, cyclotron resonance

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1. Introduction

[2] The sources and losses of the energetic radiation belt particles have been a central concern of magnetospheric physicists since the discovery of the trapped radiation populating the near space environment of our planet. One of the unexpected effects of high altitude nuclear tests has been the significant modification of the energetic particle population [*Van Allen*, 1966; *McIlwain*, 1966]. Another anthropogenic effect on the radiation belts, specifically due to operational high-power VLF communication and navigation transmitters has been documented since the mid 1970s, and includes observations of monoenergetic electrons (with energy corresponding to specific VLF transmitter frequencies) in the drift loss cone [*Vampola and Kuck*, 1978], and direct observation of individual bursts of electrons in the bounce loss cone exhibiting the same ON/OFF modulation pattern as the injected VLF wave [*Imhof et al.*, 1983a, 1983b; *Inan et al.*, 1985].

[3] At present, the relativistic electron population of the radiation belts is one of the most important concerns of "Space Weather," with a rapidly growing number of civilian and military assets in space being increasingly vulnerable (due to smaller spacecraft with much less protection and radiation hardening) to enhanced radiation levels [*Baker*, 2000, and references therein]. Thus, potential mitigation of enhanced radiation levels is a topic of significant interest.

[4] A comprehensive theoretical study of the loss rates of radiation belt electrons in the energy range 100–1500 keV was carried out recently by *Abel and Thorne* [1998a, 1998b], who separately considered the effects of different types of whistler mode waves as well as Coulomb collisions on L-shells ranging from $L \simeq 1.2$ to $L = 4$. One of their rather surprising conclusions was that VLF transmitters, operating continuously in the 17–23 kHz frequency range at various locations around the world, dominantly affect the lifetimes of electrons at $L < \sim 2.6$. Simple estimates (see below) indicate that the total effective power input into the magnetosphere by all the VLF transmitters considered in their study was ~73 kW. Thus, it should in principle be possible to produce effects of similar order by spaceborne transmitter(s) which can radiate whistler mode waves with total integrated (over spatial regions) power levels of tens of kilowatts at the appropriate frequencies and magnetospheric locations. The power requirements may actually be substantially lowered via the use of lower (<10 kHz) frequencies, due to the storage (via multiple reflections) of VLF wave energy in the magnetospheric cavity, as described below.

[5] Our purpose in this paper is to quantify the potential control of trapped radiation levels using satellite-based ELF/VLF transmitters, by providing estimates of the potential effects of a single transmitter operating at the magnetic equatorial plane at $L = 2$, using frequencies in the few kHz

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range. For quantitative examples, we focus our attention on 1.5 MeV and 3 MeV electrons.

2. Energetic Electron Lifetimes

[6] We base our estimates of the effects of a spaceborne transmitter on the lifetimes of energetic electrons on the work of *Abel and Thorne* [1998a, 1998b]. These authors considered the effects of cyclotron resonant interactions with naturally occurring whistler mode waves such as plasmaspheric hiss (generated in situ) [*Lyons et al.*, 1972; *Lyons and Thorne*, 1973; *Thorne et al.*, 1973] and externally injected whistler waves generated by lightning discharges, as well as man-made signals injected from ground-based VLF transmitters. The authors used a bounce-averaged diffusion-based formulation, accounting for resonant interactions (including zeroth, first and higher order resonances) at all points during the motion of the trapped particles along the field lines, and considering different types of waves by means of their known frequencies, rates of occurrence, power levels, and propagation characteristics. They presented their results in terms of lifetime versus L-shell, identifying the incremental contribution of the different waves and comparing the overall results with the trapped electron lifetimes measured following the Starfish upper atmospheric nuclear detonation [*Van Allen*, 1966; *McIlwain*, 1966].

[7] At $L = 2$, *Abel and Thorne* [1998a] found that the lifetimes of 500 keV electrons in the absence of any wave interactions (i.e., as determined only by Coulomb scattering interactions with the upper atmosphere near the mirror points) was $\sim 5 \times 10^3$ days. The lifetime of 500 keV electrons at $L = 2$ was found to be unaffected by plasmaspheric hiss, while scattering by lightning-generated whistlers lowered it to $\sim 3 \times 10^3$ days, still much larger than the measured lifetimes of $\sim$20 days. The estimated lifetime with VLF transmitters also included was found to be $\sim$60 days, in much better agreement with the measured lifetime. The implication is that the VLF transmitters were clearly the dominant factor for the loss of 500 keV electrons at $L = 2$. For 1.5 MeV electrons, Coulomb collisions interactions with plasmaspheric hiss reduced the lifetime to $\sim 2 \times 10^4$ days, while lightning-whistler-induced scattering further reduced it to $\sim$700 days, and VLF transmitter interactions reduced it to $\sim$100 days, once again indicating that the latter was an important determinant for the loss rate of these electrons at $L = 2$.

2.1. Crude Estimates Based on Total Injected ELF/VLF Wave Power

[8] We can crudely quantify the potential effects of a spaceborne VLF transmitter by estimating the total VLF transmitter power that was taken to be injected into the magnetospheric medium in the *Abel and Thorne* [1998a, 1998b] study and using simple power scaling. *Abel and Thorne* [1998a, 1998b] considered eight major VLF transmitters, four operating at radiated power levels of $\sim$1 MW and the other four operating at $\sim$300 kW, and used the method described in *Inan et al.* [1984] to calculate the wave magnetic field intensities along the field lines. Most of the radiated power of a ground-based source propagates in the earth-ionosphere waveguide, and although wave energy is coupled upward to the radiation belts [*Inan et al.*, 1984], the trans-ionospheric loss (due mostly to absorption in the lower ionosphere) is $\sim$10 dB during nighttime at the frequencies ($\sim$17–23 kHz) and geomagnetic latitudes (30°–40°) of interest [*Helliwell*, 1965, Figures 3–35]. Recognizing that the trans-ionospheric coupling loss is much higher ($\sim$40 dB) during daytime, *Abel and Thorne* [1998a, 1998b] assumed the transmitters to be effective only during a $\sim$7 hour period each night and also neglected the effects of the waves at those locations where the wave frequencies f were greater than one third of the local gyrofrequency f_H (i.e., $f > 0.3 f_H$) assuming that the waves would then be damped.

[9] With various propagation, loss and other factors included, *Abel and Thorne* [1998a, 1998b] represented each VLF transmitter with an effective uniform average wave magnetic field intensity over a ±15° longitudinal region at the geomagnetic equatorial plane centered around the longitude of each transmitter. As an example, for the NAA transmitter with 1 MW radiated power [*Inan et al.*, 1984], they assumed this effective peak intensity to be $B_w \simeq 10$ pT with a radial variation as shown in Figure 1b over the L-shell range $1.3 < L < 3$. The functional form of the radial variation shown in Figure 1b is $B_w(r) = 10 \sin[\pi (r - 1)/2.1]$ pT, where r is the radial distance; this radial variation closely fits that shown for the magnetic field intensity of the NAA signal in Figure 4 of *Abel and Thorne* [1998b].

[10] Assuming an average refractive index of $n \simeq 15$ as appropriate for the VLF transmitter frequencies of 17–23 kHz (actual values for n are $n = 10$, 15 and 20 at $L = 1.5$, 2.0, and 2.5 for a typical [*Carpenter and Anderson*, 1992] plasmaspheric electron density profile having a cold plasma density at $L = 2$ of $N_{eq} = 2850$ cm^{-3}) and for a wave normal angle of $\sim$45° as adopted by *Abel and Thorne* [1998a, 1998b], the total effective wave power projected onto the ±15° sector shown in Figure 1 by the NAA transmitter can be found by integrating the radial variation given in Figure 1b over the region for which $f < 0.3 f_H$, i.e.,

$$P_{\rm NAA} \simeq \left(\frac{\pi}{6}\right) \int_{1.3R_E}^{2.6R_E} \frac{c[B_w(r)]^2}{2n\mu_0} r\,dr \simeq 37 \text{ kW}$$

where μ_0 is the permeability of free space and c is the speed of light. The $\pi/6$ factor results from the fact that the wave power is assumed to be confined to the ±15° sector and uniformly distributed over this range. For the purpose of this first-order estimate, it is entirely sufficient to take the wave power density to be simply proportional to the square of the wave magnetic field intensity (i.e., time average power given by $P = (1/2)|E_w||H_w| = (1/2)(cB_w/n)(B_w/\mu_0) = [B_w^2/(2n\mu_0)]$) in the same manner as that for a uniform plane electromagnetic wave, rather than using a more complete expression for the Poynting flux of an obliquely propagating whistler mode wave, such as that given in Equation (48) of *Bell* [1984]. Considering four transmitters operating at 1 MW and the other four at 0.3 MW, each illuminating a 30° sector and operating only 7 hours per day, the total global continuous time-average VLF wave power projected by all eight transmitters onto the $1.3 < L < 2.6$ shell is

$$P_{\rm tot} = [4(1) + 4(0.3)]\left(\frac{7}{24}\right)(P_{\rm NAA}) \simeq 56 \text{ kW}$$

using $P_{\rm NAA} = 37$ kW as determined above. The diffusion coefficients used by *Abel and Thorne* [1998a, 1998b] are

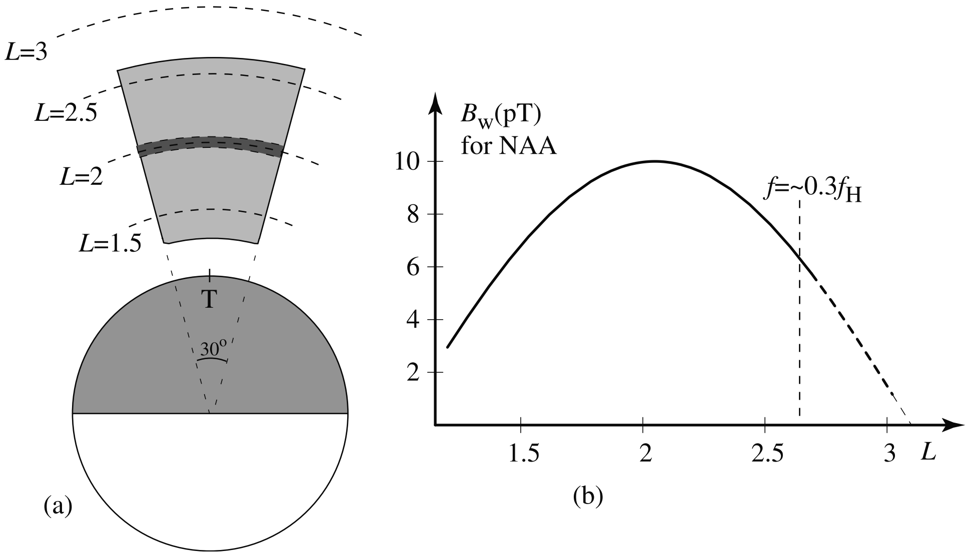

Figure 1. Schematic illustration of the magnetospheric region of influence for a ground-based VLF transmitter as used in the work of *Abel and Thorne* [1998a, 1998b]. Based on considerations of various propagation, loss and other factors, *Abel and Thorne* [1998a, 1998b] chose to represent each VLF transmitter in terms of an equatorial wave magnetic field intensity of ~10 pT over the region shown in (a), extending from L = 1.3 (below which Coulumb collisions were found to be dominant) to L = 2.6, above which the 17–23 kHz signals were above $\sim 0.3 f_H$ and thus were assumed to be damped. The functional form of the radial variation shown is $B_w(r) = 10 \sin[\pi\ (r - 1)/2.1]$ pT (where r is the radial distance), representing a close fit to that shown for the magnetic field intensity of the NAA signal in Figure 4 of *Abel and Thorne* [1998b]. (b) The corresponding meridional cross section view of the same region.

proportional to B_w^2, so that the global energetic electron lifetimes (calculated in the manner described in section 3 of *Abel and Thorne* [1998a]) are inversely proportional to this global average wave power level. For later reference, we note that the total global time-average power projected onto a narrow L-shell strip, for example within $1.95 < L < 2.05$ as shown with darker shading in Figure 1a and as in the example case discussed in the next section, can be obtained using the same expression for P_{NAA} given above but simply integrating over the narrow radial range of 1.95 to 2.05. Using the radial variation of $B_w(r)$ as given in Figure 1b, we find the power projected in this strip to be ~6 kW.

[11] On the basis of the above, it appears that at those L-shells where *Abel and Thorne* [1998a, 1998b] found the VLF transmitters to be the dominant means of loss, the particle lifetimes may be halved by doubling the effective average power, for example by using a spaceborne ELF/VLF transmitter to inject ~56 kW of wave energy in the 17–23 kHz range into the L-shell range of $1.3 < L < 2.6$. The same result could also be obtained by using 11 spaceborne transmitters distributed in longitude and radial distance, each injecting ~5 kW. Note that in view of the cumulative nature of the wave-induced diffusion process, the fact that the diffusion coefficient is proportional to B_w^2 or wave power spectral density (in units of W-m^{-2}-Hz^{-1})), and the fact that the energetic particles drift rapidly in longitude, it should not matter that the power injected by the spaceborne transmitter would necessarily be confined to a limited longitude range (rather than uniformly distributed in azimuth). The manner in which the wave power injected in situ at a fixed point is distributed in L-shell would be determined by the disposition of the whistler mode ray paths, depending on the wave frequency, the initial wave normal angle with which the waves are injected and the gradients of the cold plasma density and the ambient static magnetic field. If the injection wave normal angle and the wave frequency are such that (see Figure 3) most of the injected power remains confined to the narrow range of $1.95 < L < 2.05$, halving the lifetime of electrons in that range of L-shells would only require a single satellite injecting a total of ~6 kW.

[12] We note that the above discussion is rather crude, as it is based only on the total wave power, and overlooks other aspects (see next subsection) of the cyclotron resonance pitch angle scattering process, such as the dependence of the efficiency of the interaction on the wave normal angle and wave frequency. Nevertheless, we see that a significant degree of control of particle lifetimes can be brought about by in situ injection of modest ELF/VLF wave power levels.

2.2. Cyclotron Resonant Pitch Angle Scattering

[13] Cyclotron resonance between electrons and ELF/VLF whistler mode plasma waves occurs when the Doppler shifted wave frequency seen in the frame of the energetic electrons is approximately equal to a harmonic of the gyrofrequency of the electrons, in accordance with the general condition:

$$\omega - k_{\parallel} v_{\parallel} \simeq \frac{-m\omega_{\mathrm{H}}}{\gamma} \qquad (1)$$

where ω is the wave frequency, ω_{H} is the electron cyclotron frequency, $k_{\parallel} = (\omega n/c) \cos\psi$ (with n being the refractive index and ψ being the wave normal angle, i.e., the angle between the wave vector and the ambient magnetic field $\mathbf{B}_0$) and $v_{\|}$ are the components of the wave vector and the particle velocity along the ambient magnetic field, $\gamma = (1 - v^2/c^2)^{-1/2}$ is the relativistic factor with v and c being respectively the particle velocity and speed of light in free space, and $m = 0, \pm 1, \pm 2\ldots$ is the harmonic resonance number. For whistler mode waves, with wave frequency lying below the electron gyrofrequency ($\omega < \omega_H$) but above the proton gyrofrequency, resonance occurs with counter-streaming electrons (i.e., $v_{\|}$ directed opposite to $k_{\|}$) for $m < 0$ and costreaming electrons for $m \geq 0$. The refractive index n is generally a function of ω, ω_H, the electron plasma frequency ω_p, and ψ, as given by the well-known Appleton-Hartree equation [*Helliwell*, 1965, p. 23]. Whistler mode waves are "slow" electromagnetic waves with refractive indices $n \gg 1$, typically in the range $\sim$10 to 100, but which can be higher (in which case the waves become quasi-electrostatic and $n_{\|}$ is bounded by thermal effects to $<\sim$1000) for wave normal angles ψ approaching the resonance cone angle ψ_r [*Budden*, 1985, p. 105].

[14] The case of $m = 0$ is also referred to as the Landau or longitudinal resonance and is qualitatively different from the resonances for $m \neq 0$. Typically for cyclotron resonances ($m \neq 0$), the primary wave force which acts on high energy particles is the $q_e\mathbf{v} \times \mathbf{B}_w$ force, which changes the direction of momentum (i.e., the pitch angle) of the electrons with negligible change (due to the $q_e\mathbf{E}_w$ force) in the particle energy [*Inan et al.*, 1978]. The cumulative effect of the wave forces over the spatial region within which (1) is satisfied reduces the pitch angle of some of the energetic electrons, moving them from higher to lower pitch angles, sending them into the loss cone if the particles are initially near its edge. Other electrons have their pitch angles increased.

[15] The physical nature of the wave-particle interaction process is qualitatively different for coherent versus incoherent waves [*Inan*, 1987]. The trajectories followed by particles when interacting with incoherent waves represent a random walk in velocity space, while for coherent waves the pitch angle and velocity of the particles vary in a well-defined manner that nevertheless depends on gyrophase. Thus, when considering the interaction of an ensemble of electrons (with different inital velocities and pitch angles) even coherent wave interactions can be described in terms of "diffusion" in velocity space [*Inan*, 1987; *Albert*, 2000]. This is especially true for interactions involving obliquely propagating waves (i.e., $\psi \neq 0$), for which higher order resonances other than the fundamental first-order counter-streaming ($m = -1$) resonance are also possible, and cumulatively contribute to the net overall scattering of a given particle during its bounce motion, each resonance being effective in a different region along the field line. The net result of the wave-induced diffusion process is that the energetic electrons execute a random walk towards the loss cone and are precipitated into the atmosphere if they reach the loss cone.

[16] For both coherent and incoherent wave-particle interactions, the magnitude of the "diffusion" coefficient (which is proportional to the ensemble average of the square of the pitch angle change, i.e., $\langle(\Delta\alpha)^2\rangle$, and is a measure of the efficiency of the pitch angle scattering) is proportional to B_w^2 [*Inan*, 1987]. Also important is the "interaction time," i.e., the duration of time over which (1) is satisfied; the longer the interaction time, the larger is the cumulative pitch angle changes. This interaction time T_r (or resonance duration) can be determined on the basis of a change in phase ϕ (the angle between $v_\perp$ and $\mathbf{B}_w$; note that $[\partial\phi/\partial t] = 0$ when (1) is exactly satisfied) of $\Delta\phi \simeq \pi$ [*Inan et al.*, 1983]. For coherent interactions near the geomagnetic equator, one finds [*Inan*, 1987] $T_r \simeq [16\pi R^2/(9\omega_H v_{\|}^2)]^{1/3}$, where R is the radial distance. Such analytical expressions are only of limited use and are only cited here for information, since numerical evaluations of pitch angle scattering coefficients [e.g., *Albert*, 2001] inherently account for the resonance time. In general, it should be noted that the interaction time can also be affected by the morphology of the background magnetic field since it essentially depends on the field line curvature near the equator. There are many processes that can affect this curvature, such as the magnitude of the ring current, or large-scale pulsations, even at relatively low L values [*Friedel and Hughes*, 1993].

[17] Figure 2a shows the resonant wave frequency as a function of geomagnetic latitude for different cyclotron harmonic resonances respectively with 1500 keV electrons initially near the loss cone ($\sim$16.8°) at $L = 2$. The cold plasma density was assumed to vary in accordance with diffusive equilibrium, with an equatorial cold plasma density of $N_{eq} = 2850$ cm^{-3} at $L = 2$ [*Carpenter and Anderson*, 1992]. The curves shown are for a wave normal angle of $\psi = 45°$, which was the value adopted by *Abel and Thorne* [1998a, 1998b] in their determinations of the effects of VLF transmitters. The general trend is that the resonant frequency for fixed particle energy increases with increasing harmonic number m, and also varies more rapidly with latitude for higher m. In general, the lowest order counter-streaming gyroresonance interaction (i.e., $m = -1$) is the most efficient for electron scattering by whistler mode waves with $\omega < \omega_H$, as illustrated in Figure 2b, which shows the bounce-averaged pitch angle diffusion coefficient $\langle D_{\alpha\alpha}\rangle$ as a function of resonance order m for 1500 keV electrons at $L = 2$ and for wave frequencies of $f = 20$, 5, 2.5, and 1.3 kHz. For the results shown in Figures 2b and 2c, the full cold plasma dispersion relation was used in conjunction with the detailed diffusion coefficient of *Lyons* [1974] (with relativistic extension, as in *Albert* [1999]), evaluated in the limit of vanishing width of the frequency and wave normal angle distributions as in *Albert* [2001]. Our evaluation of $\langle D_{\alpha\alpha}\rangle$ differed somewhat from the procedure of *Abel and Thorne* [1998a, 1998b], who used the simplified expressions of *Lyons et al.* [1972]. The wave normal angle ψ and cold plasma density N_{eq} were taken respectively to be 45° and 2850 cm^{-3} to correspond to those used for Figure 2a, the wave power density was kept constant for the different frequencies and was taken to correspond to a wave magnetic field intensity of 30 pT at the geomagnetic equator at $\sim$2.5 kHz.

[18] Another result evident in Figure 2b is the fact that $\langle D_{\alpha\alpha}\rangle$ is higher for lower frequencies. This behavior is further illustrated in Figure 2c which shows $\langle D_{\alpha\alpha}\rangle$ as a function of frequency for the first-order resonance ($m = -1$). The fundamental reason for the higher values of $\langle D_{\alpha\alpha}\rangle$ at lower frequencies can be seen upon examination of the behavior of the resonant frequency curves in Figure 2a. It is

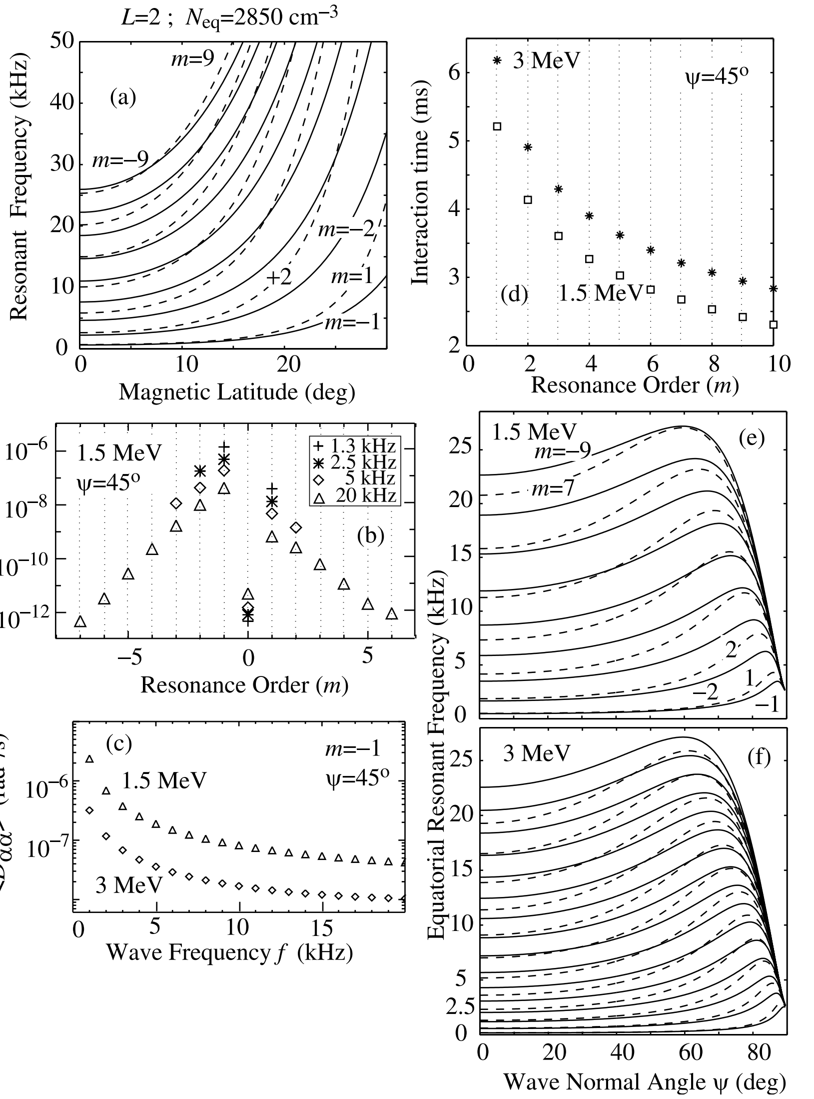

Figure 2. (a) Resonant frequency versus geomagnetic latitude at L = 2 for 1500 keV electrons for different cyclotron harmonic resonances, for a wave normal angle of $\psi = 45°$. (b) Bounce averaged diffusion coefficient for different harmonic resonances at four selected frequencies. The wave power densities were taken to be the same in each case, corresponding to wave magnetic field intensity of 30 pT at the equator. (c) Bounce averaged diffusion coefficient for the first-order counterstreaming resonance shown as a function of wave frequency for 1.5 MeV and 3 MeV electrons. (d) Interaction time for near equatorial interactions for 1.5 MeV and 3 MeV electrons for different cyclotron harmonic resonances. (e) Resonant frequency for first-order gyroresonance (m = 1) with 1500 keV electrons as a function of the wave normal angle ψ. (f) Same as (e) but for 3 MeV electrons.

evident that the lower the wave frequency the closer is the point of resonance (i.e., intersection of the horizontal line at that frequency with the $m = -1$ curve) to the geomagnetic equator, providing for slower variation of resonant frequency with geomagnetic latitude (i.e., longer interaction or resonance times). As a result, waves at a fixed frequency can stay in resonance longer with the 1.5 MeV particles, leading to a larger bounce averaged diffusion coefficient. A similar physical effect is illustrated in Figure 2d, which shows the interaction time (calculated assuming a coherent wave and using a $|\Delta\phi| < \pi$ criterion) as a function of resonance order m. For this plot, the wave frequency for each resonance m was taken to be that which is exactly resonant at the equator. It is clear the interaction time (and thus the total scattering, noting that $\langle(\Delta\alpha)^2\rangle$ is proportional to $D_{\alpha\alpha}T_r$, where $D_{\alpha\alpha}$ is the local (i.e., not bounce-averaged) pitch angle diffusion coefficient is higher for the lower order resonances, resulting from the fact that the corresponding resonance curves in Figure 2a vary slower with geomagnetic latitude.

[19] On the basis of the result from Figure 2b that $\langle D_{\alpha\alpha}\rangle$ for 2.5 kHz is a factor of ~30 higher than that for 20 kHz, we can conclude, once again based on simple power scaling, that the lifetimes of 1500 keV electrons may be halved by using a spaceborne ELF/VLF transmitter to inject ~(56 kW/30) = 1.8 kW of wave energy in the few kHz range into the L-shell range of $1.3 < L < 2.6$. In other words, waves at few kHz frequencies would drive a bounce-

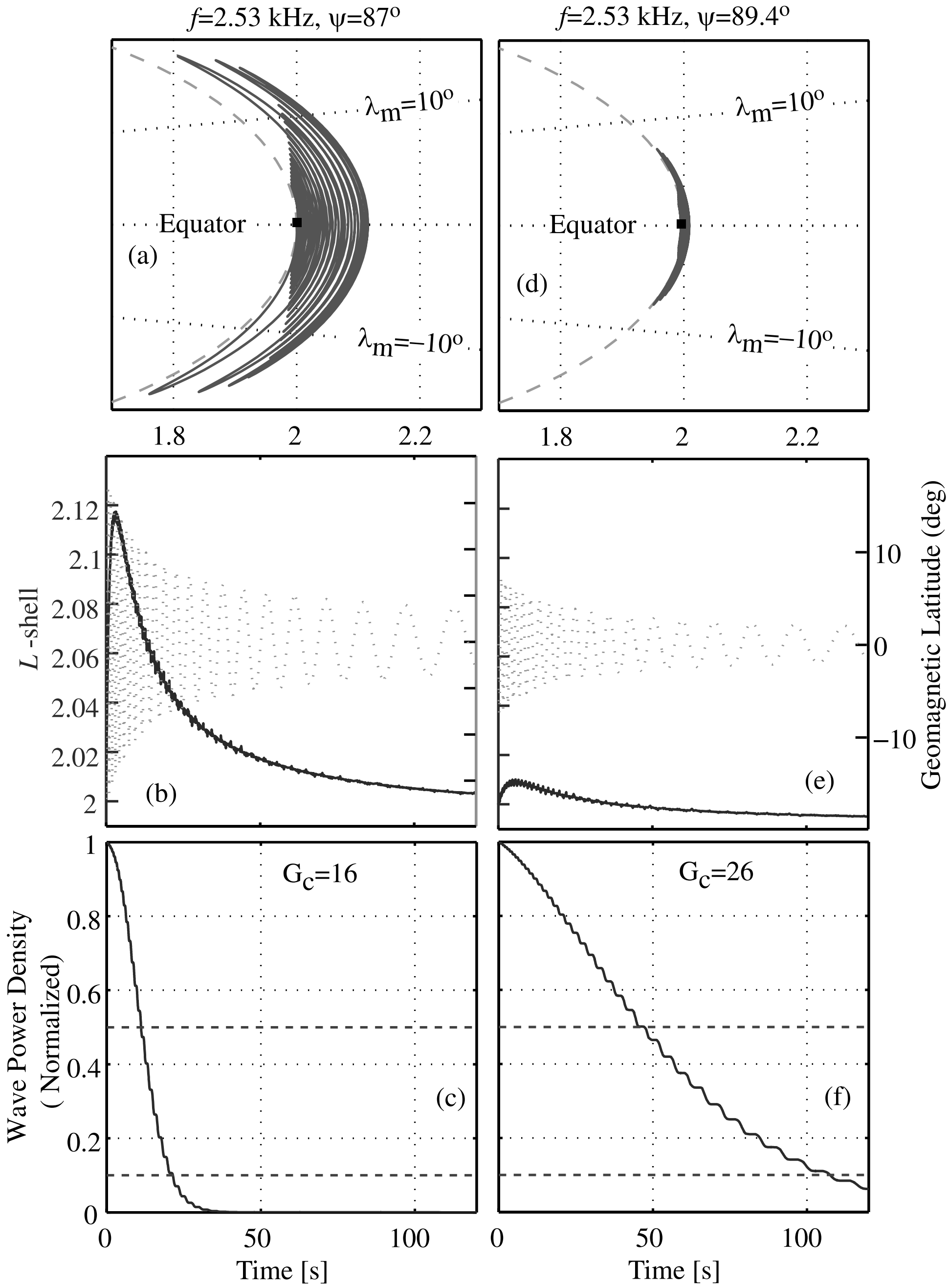

Figure 3. (a) Ray path of a 2.53 kHz signal injected at the equator at $L = 2$ at $\psi = 87°$. (b) Latitude and L-shell versus time along the ray path. (c) Variation of the wave power density (attenuated due to Landau damping) along the ray path. (d) Same as (a) but for $\psi = 89.4°$. (e) Same as (b) but for $\psi = 89.4°$. (f) Same as (c) but for $\psi = 89.4°$.

averaged diffusion coefficient of more than an order of magnitude higher than the 17–23 kHz waves considered by *Abel and Thorne* [1998a, 1998b], so that 1.8 kW of total power at a few kHz would have the same effect on particle lifetimes as 56 kW at 17–23 kHz. If the injection wave normal angle and the wave frequency are such that (see Figure 3) most of the injected power remains confined to the narrow range of $1.95 < L < 2.05$, halving the lifetime of electrons in that range of L-shells would only require a single satellite injecting a total power (at a few kHz frequencies) of one thirtieth of ∼6 kW or ∼200 Watts, assuming that $\langle(\Delta\alpha)^2\rangle$ does not vary significantly during this time.

[20] Our conclusion from Figure 1b that few kHz waves are more efficient that ∼20 kHz waves might appear in conflict with the *Abel and Thorne* [1998a, 1998b] conclusion that (for $L < \sim 2.6$) the ∼20 kHz VLF transmitter signals are more important in terms of their net effect on particle lifetimes than few kHz whistlers (injected by lightning). However, their overall result was due to a combination of factors, such as the effective "occurrence rate" which they assumed for six VLF transmitters amounts to ∼12% as

opposed to only ~3% for whistlers and also the assumption of much higher bandwidth (±2 kHz) for whistlers versus the ±50 Hz bandwidth of VLF transmitter signals. All other factors being the same, it is clear from Figure 1b that few kHz waves are more efficient scatterers.

[21] On the basis of Figure 2a, 17–23 kHz VLF transmitter signals considered by *Abel and Thorne* [1998a, 1998b] could interact with 1500 keV electrons at $L = 2$ in harmonic resonances of order $m = \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, +7$ and +8. However, as argued above and shown in Figure 2b, the most significant resonances (i.e., those which have the highest bounce-averaged diffusion coefficient) are the lowest order ones. For a wave frequency of 20 kHz, the $\langle D_{\alpha\alpha} \rangle$ decreases by nearly an order of magnitude for each successive increment in m for the counterstreaming resonances ($m < 0$), and is even lower for $m > 0$. Note that $\langle D_{\alpha\alpha} \rangle$ is miniscule for the Landau resonance ($m = 0$) since this interaction occurs largely via the longitudinal (i.e., along the magnetic field line) component of the wave electric field, which is relatively small except when the wave normal angle ψ is near the resonance cone, when the refractive index is high and the whistler mode wave is largely electrostatic.

[22] While the ground-based VLF transmitter frequencies were fixed to be operating in the 17–23 kHz range, a dedicated spaceborne ELF/VLF transmitter can in principle operate at a frequency chosen specifically to maximally scatter the targeted electrons (in our case the 1500 keV and 3000 keV) at the desired location (in this case $L = 2$). The choice of such optimum operational frequencies would depend on considerations of the overall diffusion rates and lifetimes of particles at all pitch angles and in the energy ranges of interest [*Albert and Inan*, 2001]. For example, it is clear from Figure 2b and Figure 2c that for scattering 1.5 MeV electrons, utilization of lower frequencies in the few kHz range would be substantially better than those in the 17–20 kHz range. The choice of operational frequency may also depend on other factors, such as the opportunity to excite the magnetospheric cavity (see next section) within which waves may endure for many tens of seconds, continuing to scatter particles. By simply operating at a more favorable frequency, a spaceborne system can drive a diffusion coefficient which is a factor of ~30 higher than that due to 17–23 kHz waves, with proportional reductions in particle lifetimes.

[23] Another possible means by which interaction length (and thus scattering efficiency) can be enhanced is evident upon examination of the resonant frequency versus latitude plots of Figures 2a and 2b. In principle, a spaceborne ELF/VLF source, for example located at the geomagnetic equator, can transmit a chirped signal, the frequency-time variation of which compensates for the variation of ω_H and $k_{\|}$, thus extending the spatial extent of the region over which (1) is valid. On the basis of Figures 2a and 2b, and for a wave propagating away from the equator, it would be desirable to have the wave frequency increase with time, so that it can stay in resonance with 500 keV electrons as it propagates to regions of lower resonance frequency. Chirped modulations have been previously discussed and were successfully employed in ground-based VLF wave-injection experiments [*Brinca*, 1981; *Helliwell et al.*, 1990], leading to enhanced rapid wave-growth and emission triggering. On the other hand, such coherent modulation schemes may scatter some particles more at the expense of others which are scattered less. In this connection, it has been shown that the resultant pitch angle scattering of particles over a range of energies may be the same for coherent versus incoherent hiss-like signals [*Inan*, 1987; *Albert*, 2001].

2.3. Wave Energy Buildup due to Magnetospheric Reflections

[24] The potential ability of a spaceborne VLF transmitter to affect the lifetimes of energetic particles can be significantly leveraged by the fact that the magnetosphere behaves like a resonant ELF/VLF cavity, with whistler mode waves magnetospherically reflecting multiple times at points where the wave frequency is approximately equal to the lower hybrid frequency [*Edgar*, 1976], leading to a buildup of wave energy available to drive particle diffusion. To quantitatively illustrate this concept, we continue to focus our attention at $L = 2$, and consider the in situ injection of a 2.53 kHz wave with an initial wave normal angle of $\psi = 87°$ pointing radially outward, at the geomagnetic equator. Figures 3a and 3b show the signal ray path as determined using the Stanford 2-D raytracing program [*Inan and Bell*, 1977], indicating that the wave energy reflects back and forth between hemispheres, remaining confined to a latitude range of ±15° and an L-shell range of $\sim 0.1L$.

[25] Satellite observations of magnetospherically reflected whistlers [*Smith and Angerami*, 1968; *Edgar*, 1976; *Gurnett and Inan*, 1988] indicate that up to 20 to 40 magnetospheric reflections can at times occur. In the largely collisionless magnetospheric medium, the primary means of attenuation of such highly oblique whistler mode waves is Landau damping due to suprathermal electrons. To quantitatively determine the extent to which the waves are Landau damped, we use the formulation of *Brinca* [1981], involving the determination of the imaginary part of the refractive index due to a specified distribution of suprathermal electrons. We use an electron distribution of $f(v) = 2 \times 10^5 v^{-4}$ cm^{-6} -s^3 as an approximate fit to recent measurements [*Bell et al.*, 2002] of 100 eV to 1 keV electrons with the HYDRA instrument on the Polar spacecraft [*Scudder et al.*, 1995]. The wave power density as a function of time, calculated in this way is shown in Figure 3c. The power density falls below ~5% of its initial value after ~28 crossings of the magnetic equator with decreasing intensity as a function of time. The "effective" number of equatorial crossings (or wave bounces) at the initial wave power level (determined by weighting each crossing with the wave power density) is ~16, meaning that the whistler mode wave would be ~16 times more effective in scattering electrons. This magnetospheric cavity enhancement "gain" is denoted by G_c and can be viewed as an effective amplification of the wave power density by the same factor, with a consequent resultant enhancement in the time-integrated diffusion and a resultant reduction in particle lifetimes. For the case illustrated in Figure 3, the cavity enhancement gain is $G_c \simeq 16$.

[26] Once again we can power scale from the results of *Abel and Thorne* [1998a, 1998b] to come up with initial estimates. Note that the total global average VLF transmitter power level used by them for the strip of $1.95 < L < 2.05$

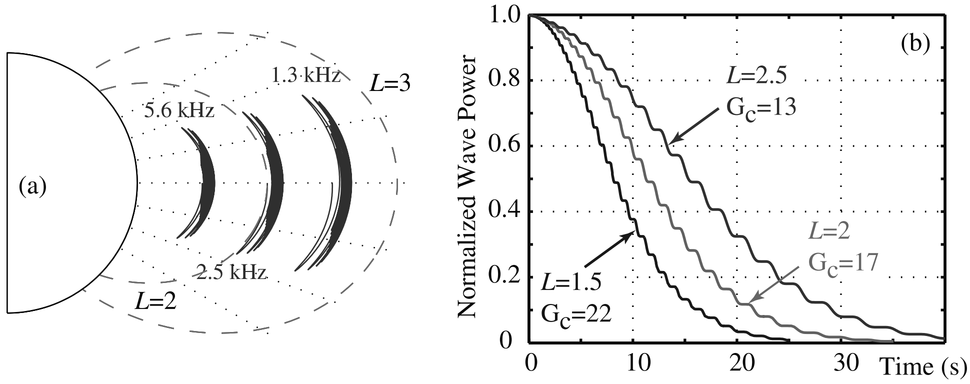

Figure 4. (a) Ray paths of 5.6, 2.53, and 1.33 kHz signals injected at the equator respectively at L = 1.5, 2.0, and 2.5 and ψ = 86.4°, 87.4°, and 87.7°. (b) Variations of the wave power density (attenuated due to Landau damping) along the ray paths at different L-shells.

was ~6 kW at 17-23 kHZ range of frequencies. In the previous section we found that an equivalent amount of lifetime reduction can be affected with only ~200W if we use few kHz frequencies which can drive higher bounce-averaged diffusion coefficients. With the cavity enhancement factor recognized as an additional factor, we can further conclude that a spaceborne transmitter located at the equator at L = 2 would need to radiate a total power of possibly as little as (200W/16) $\simeq$ 13 W at 2.53 kHz in order to reduce the lifetimes of 1500 keV electrons (within such a narrow L-shell range illuminated by the wave energy) by a factor of 2 (i.e., affect lifetimes as much as all of the ground-based VLF transmitters), as long as this power is emitted at a wave normal angle of ~87°. Radiation properties of typical electric dipole and loop antennas are discussed in the next section, but we note in passing that at VLF frequencies and for antennas of reasonable size ($\leq$100 m), most of the wave energy is in fact emitted at these high angles [*Wang and Bell*, 1972a].

[27] The quality factor of the magnetospheric cavity, quantified with the gain factor G_c depends on a number of parameters. In general, G_c is higher (i.e., injected waves endure longer) when the initial wave normal angle ψ is closer to the resonance cone. This behavior is illustrated in the second column of Figure 3, where we show results for the same parameters as in the first column but for an initial wave normal angle of ψ = 89.4°. We see (Figures 2d and 2e) that the wave remains confined to a narrower range in both latitude (±7°) and L-shell (±0.01L) and the wave energy endures much longer (Figure 3f), resulting in $G_c \simeq 26$.

[28] Effective excitation of the magnetospheric cavity also depends on wave frequency. Specifically, for a source located at the geomagnetic equator, optimum excitation (i.e., longest endurance of injected waves) is achieved by launching waves at a frequency slightly below the equatorial lower hybrid resonance frequency (f_{LHR}). Accordingly, the wave frequency for optimum number of magnetospheric reflections is different at each L-shell. To illustrate this point, Figure 4 shows multiply reflecting rays injected at L = 1.5, 2.0, and 2.5, respectively at wave normal angles of 87° and frequencies of 5.6 kHz, 2.53 kHz, and 1.33 kHz. We see from Figure 4a that the behavior of the ray paths are similar, remaining confined to narrow regions of ~0.1L in extent. The gradual decrease of the wave power densities (due to Landau damping) are shown in Figure 4b. Noting that the physical path lengths traveled by the waves in each bounce are different and the wave velocities are frequency dependent, the damping rates shown represent effective factors of wave power enhancements of $G_c \simeq 22$, $G_c \simeq 16$, and $G_c \simeq 13$ respectively at L = 1.5, 2.0, and 2.5. Note here that although the 5.6 kHz wave at L = 1.5 damps away faster than the 2.53 kHz wave at L = 2, the number of equatorial crossings (and thus the cavity gain G_c which is defined on this basis) is still higher, due to the shorter ray path between hemispheric reflection points.

[29] The ability of the magnetospheric cavity to store magnetospherically reflecting whistler mode waves is a relatively robust property which is not critically dependent on the initial parameters. Deviations from the requirements of excitation at ($f \simeq f_{LHR}$) and ψ near ψ_r result in the wave energy being spread out to different L-shells, and reduced values of G_c, but in a gradual manner. This point is illustrated in Figure 5, which shows that injection at $\psi = -70°$ at L = 2 and at f = 2.53 kHz results in $G_c \simeq 8$ and that injection at f = 1 kHz gives $G_c \simeq 10$. Both of these gain factors are still considerably high, reducing the wave power requirements for controlled precipitation by an order of magnitude. In general, the fact that the wave power is dispersed over a broader range of L-shells is not detrimental and is in fact consistent with the overall objective of radiation belt control, since in practice one would want to affect more than just a limited L-shell region. If the wave power injected at a given L-shell is dispersed so much that each particle essentially interacts only once with the wave, the diffusion coefficient for each particle would be much smaller, resulting in longer lifetimes, although the net loss in terms of number of particles affected (considering the full range of L-shells over which the wave power is dispersed) would still be the same as in the cases of wave energy confined to a narrow L-shell range. In practice, however, one would probably want to induce controlled precipitation over a range of L-shells, using VLF signal sources distributed over the range L-shells. Accordingly, whether wave energy injected at one L-shell remains at that L-shell or is dispersed should not matter, since wave energy dispersed to other L-shells would be compensated by waves injected at those L-shells spilling over the L-shell of interest. Quantitative determination of the resultant cavity gain G_c under

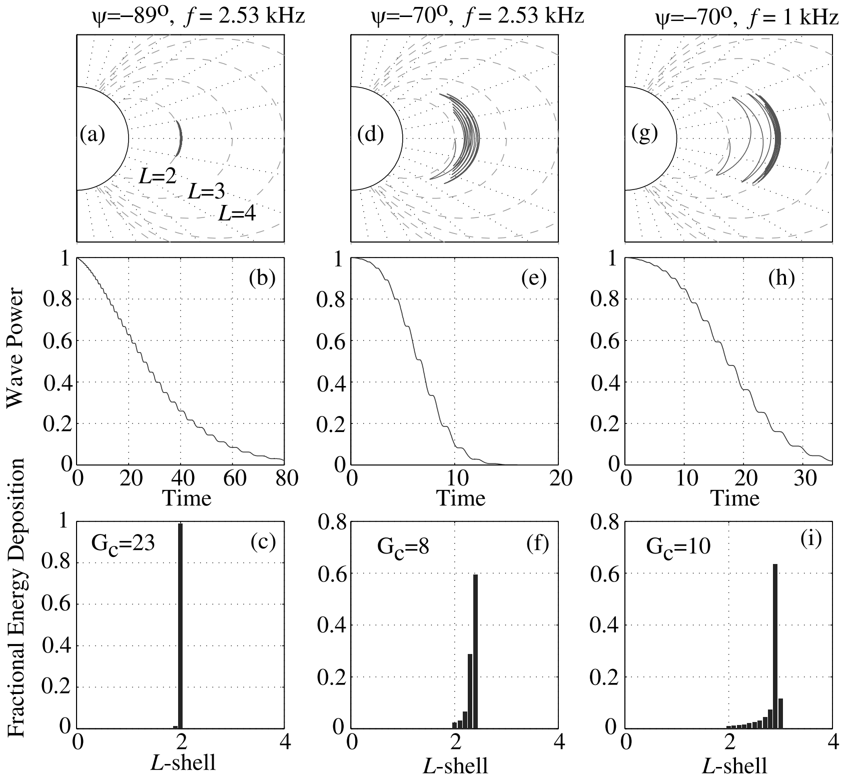

Figure 5. (a) Ray path of a 2.53 kHz (just above the local lower hybrid frequency) signal injected at the equator at $L = 2$ with a wave normal angle of 89°. Note that the wave energy remains closely confined to the original L-shell. (b) Variation of the wave power density (attenuated due to Landau damping) along the ray path. The wave energy persists for ~80 seconds, crossing the equator multiple times until its power diminishes. (c) Fractional deposition of wave energy (the fractional time spent by the ray path) as a function of L-shell. (d) Ray path for a wave injected at a wave normal angle of 70°. (e) Wave energy persists for a much shorter time, i.e., ~10 sec. (f) Energy deposition occurs over a broader range of L-shells. (g) Radiation at the same point at frequencies below the local lower hybrid resonance frequency (1 kHz shown as an example) causes the ray path to disperse to higher L-shells. (h) Wave energy does in fact persist for up to ~25 to 30 seconds, and (i) illuminates a relatively broad range of L-shells. Note that the maximally illuminated L-shell is now higher, close to $L = 3$, as opposed to $L = 2$ for $f = 2.53$ kHz.

such conditions must await the formulation of global models of wave-driven diffusion which fully account for magnetospheric reflections and the wave normal spectrum of VLF waves injected in situ, a task well beyond the scope of this preliminary paper.

2.4. Dependence of Wave-Driven Diffusion on Wave Normal Angle ψ

[30] We note that the diffusion calculations reported by *Abel and Thorne* [1998a, 1998b] assumed a wave normal angle of $\psi = 45°$, whereas the k-vectors for the magnetospherically reflecting waves shown in Figures 3, 4, and 5 are at higher angles closer to the resonance cone. The variation of the equatorial resonant frequency with wave normal angle ψ is shown in Figures 2e and 2f, respectively for 1.5 MeV and 3 Mev electrons. As the wave normal angle is increased we can see from (1) that the resonant frequency for any given harmonic (i.e., constant m) first increases due to the fact that increasing ψ reduces $k_{\|}$. However, as we approach the resonance cone, the whistler mode refractive index (and thus the magnitude of the wave number **k**) increases and dominates over the cos ψ factor. While Figures 2e and 2f show the equatorial resonant frequency, we note that the resonant frequency for any given harmonic increases with distance away from the equator, in a manner illustrated in Figure 2a for $\psi = 45°$. Although it is clear that waves at frequencies of a few kHz can resonantly interact with highly oblique waves, we need to note that the pitch angle diffusion coefficient for energetic electrons resonantly interacting with whistler mode waves does depend on ψ [e.g., see *Abel and Thorne*, 1998b]. Generally, $\langle D_{\alpha\alpha} \rangle$ for any given resonance remains steady up to a given ψ and then decreases either due to the fact that the frequency of choice falls below the resonant frequency (Figure 2e) or due to the particular variations of the Bessel function terms inherent in the basic quasi-linear diffusion coefficients [*Lyons et al.*, 1972]. Such terms represent the lack of alignment of the plane of gyration of the electrons and the oblique wave fronts and can produce deep nulls for certain wave normal angles for a given particle pitch angle, or equivalently for certain particle pitch angles for a given wave normal angle [*Albert*, 1999]. Also, as the wave normal angle increases, the contributions of the other harmonic resonances can become comparable to that of the funda-

mental counterstreaming resonance, but once again up to a certain wave normal angle ψ beyond which they decrease due once again to the Bessel function terms. The diffusion coefficient for the longitudinal resonance ($m = 0$) increases steadily with increasing ψ (due primarily to the increased relative magnitude of the wave electric field component along the field line), and much more rapidly as ψ approaches the resonance cone, at which time the wave becomes highly electrostatic, so that scattering via the wave electric field increases. This behavior may well drive the total $\langle D_{\alpha\alpha} \rangle$ back up to high levels, so that its value in the immediate vicinity of the resonance cone may be comparable to the value at $\psi = 45°$ upon which the *Abel and Thorne* [1998a, 1998b] calculations were based. However, whether the increased scattering due to the $m = 0$ resonance can completely compensate for the reduced scattering due to the cyclotron resonances depends on the particular parameters, so that the quantitative assessment of this dependence is beyond the scope of the present paper.

3. Radiation Properties of Electric Dipole Antennas

[31] Our discussions up to this point indicate that the lifetime of energetic radiation belt electrons may be significantly affected by in situ injection of waves in the inner belt and slot regions. In this context, it is important to review our current understanding of radiation of VLF waves from space-based antennas. In free space, a dipole antenna radiates most efficiently when the total length of the antenna is equal to one half of the wavelength of the wave radiated by the antenna. While in free space λ is not a function of the direction of wave propagation, and is a constant for any given frequency, in a magnetized plasma such as the magnetosphere, λ is a strong function of the wave normal angle ψ between the *k*-vector and the magnetic field $\mathbf{B}_0$. In this case the antenna most intensely radiates into waves for which the antenna length is comparable to $\lambda(\psi)/2$.

[32] At VLF frequencies in the inner magnetosphere the wave length in the direction of $\mathbf{B}_0$ is generally several kilometers, while in the direction of the resonance cone ($\psi \simeq \psi_r$) it can be as small as a few meters. Thus, an antenna of modest length ($\sim$100 m) operated at VLF frequencies tends to radiate most strongly into waves whose wave normals are close to the resonance cone direction, which near the equator in the inner magnetosphere (with $f \ll f_H$) is nearly perpendicular to $\mathbf{B}_0$. For example, according to *Wang and Bell* [1972a], the major lobe of the radiation pattern of a VLF dipole antenna in a magnetized plasma is produced by waves with wave normal direction $\psi_m \simeq \cos^{-1}(2f/f_H)$, with a beam width of a few degrees.

[33] In the context of the example discussed in the previous section, we focus our attention on a spaceborne transmitter located at the equator at $L = 2$ operating at 2.53 kHz. The local value of the electron gyrofrequency is $\sim$110 kHz. In this case $\psi_m = \cos^{-1}(2 \times 2.53/110) = 87.3°$, and most of the radiated VLF power will be carried by waves with $\psi \simeq 87.3°$. In particular, approximately one half of the radiated power is carried by waves with wave normals in the range $84° < \psi < \psi_m$. The radiation resistance of such an antenna would be $\sim$200–500Ω [*Wang and Bell*, 1970], with a radiation efficiency of $\sim$50–80% [*Wang and Bell*, 1972b]. Thus, a tuned electric dipole antenna with an applied voltage of 1000 V could radiate between 1.5 to 4 kW of total power, for an input transmitter power of 2 to 5 kW.

4. Summary and Discussion

[34] Simple power scaling analysis of the results of *Abel and Thorne* [1998a, 1998b] indicates that spaceborne VLF transmitter(s) radiating total whistler mode power levels of $\sim$6 to 56 kW (depending on whether the radiated wave energy is confined to a narrow *L*-shell region such as $1.95 < L < 2.05$, or spread over a wider range of *L*-shells, such as $1.3 < L < 2.6$) at frequencies of 17–23 kHz can potentially reduce the lifetimes of 1500 keV electrons near $L \simeq 2$ by a factor of two.

[35] A similarly located transmitter operating at frequencies of a few kHz may be more efficient by a factor of $\sim$30, interacting with 1500 keV electrons in first or second order (rather than higher order) cyclotron harmonic resonances. Furthermore, when the highly efficient storage of VLF waves in the magnetospheric cavity is taken into account, we conclude that continuously injected VLF wave energy builds up and is enhanced by a factor of $\sim$16 (for the specific example case considered). A combination of the two factors would suggest that the lifetimes of 1500 keV electrons within the narrow range $1.95 < L < 2.05$ may be reduced by a factor of two with a spaceborne transmitter radiating a total power of as little as $(200/16) \simeq 13$ W at $\sim$2.5 kHz.

[36] In view of the diffusive nature of the wave-induced scattering process, particle lifetimes are simply proportional to the total ELF/VLF power level, so that the same result can be achieved by using single or multiple satellite-based transmitters. Multiple transmitters may be suitably distributed in longitude or radial distance, depending on the range of *L*-shells of interest, to spread the VLF wave power in the desired manner and/or to selectively affect electrons in given energy ranges. As an example, examination of Figure 5 indicates that different magnetospheric regions can be illuminated from the same location simply by choice of operational frequency.

[37] The potential use of spaceborne ELF/VLF transmitters to control the lifetimes of radiation belt electrons may be further leveraged by the additional fact that whistler mode waves can be amplified as a result of gyroresonant interactions with energetic electrons. Based on results of coherent ELF/VLF wave-injection experiments, rapid temporal wave growth of up to 30 dB can be achieved under the right conditions, culminating with the triggering of VLF emissions [*Helliwell*, 1988]. Both the amplified and newly triggered waves would in turn pitch angle scatter the electrons, further reducing the particle lifetimes. To take full advantage of this potential free energy source requires the use of carefully selected frequencies and frequency-time formats, and a thorough understanding of the magnetospheric conditions and regions under which such wave growth and triggering is manifested.

[38] Although our results suggest that control of the electron population in the radiation belts may well be within our reach, it is important to note that the resultant quantitative effects on a global scale can only be determined in the context of global models of radiation belts which account

for acceleration and loss of energetic electrons. For example, the removal of particles by artificial injection of waves may lead to local anisotropies which may drive instabilities, producing waves which in turn may accelerate lower energy particles back up to those energies which were just successfully removed. Although such stimulated waves would in general lead to more pitch angle scattering than energy change, their generation and effects on the particles should be properly modeled before we can fully quantify the overall effects on the radiation belts of artificially injected VLF waves. Localized removal of particles may also cause local gradients which may lead to enhanced radial transport which would tend to replenish what was just removed, although radial diffusion rates are known to be much smaller than wave-driven diffusion inside the plasmasphere [*Lyons and Thorne*, 1973].

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