

Real-data Mesoscale Applications of EnKF and Towards coupling EnKF with 4DVAR

Fuqing Zhang
Texas A&M University

Contributors: Ellie Meng, Yonghui Weng, Meng Zhang, and Jim Hansen

Variational vs. Sequential Data Assimilation

Variational approach through minimizing a cost function (3DVAR, 4DVAR)

$$2J(x) = (x-x^f)^T \mathbf{B}^{-1} (x-x^f) + (y-Hx^f)^T \mathbf{R}^{-1} (y-Hx^f)$$

Sequential methods through OI or Kalman filtering (EKF, EnKF)

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}(\mathbf{y} - \mathbf{H}\mathbf{x}^f)$$

EnKF vs. 3DVAR

EnKF is essentially 3Dvar except for w/ flow-dependent \mathbf{B} estimated from ensembles

Performance: EnKF outperforms 3DVAR in most real-data comparisons

Better performance from coupling EnKF and 3DVAR, explicitly or implicitly

EnKF vs. 4DVAR

EnKF and 4DVAR are equivalent under perfect model, linear dynamics

Performance: comparable in OSSEs; EnKF slightly behind in operational systems of EMC; comparable in JMA

Comparing EnKF with 3Dvar for June 2003

Observations:

Soundings every 12 h QC'd by WRF/3Dvar in D2, assuming observational errors of NCEP.

3DVar: the default background error covariance cv3

WRF/EnKF:

Single-scheme ensemble

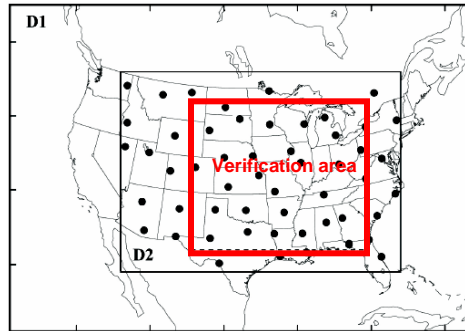
Multi-scheme ensemble

Grell
YSU PBL
WSM 6-class

Grell / KF / BM
YSU / ETA / MRF
WSM 6-class / Thompson et al. / Lin et al.

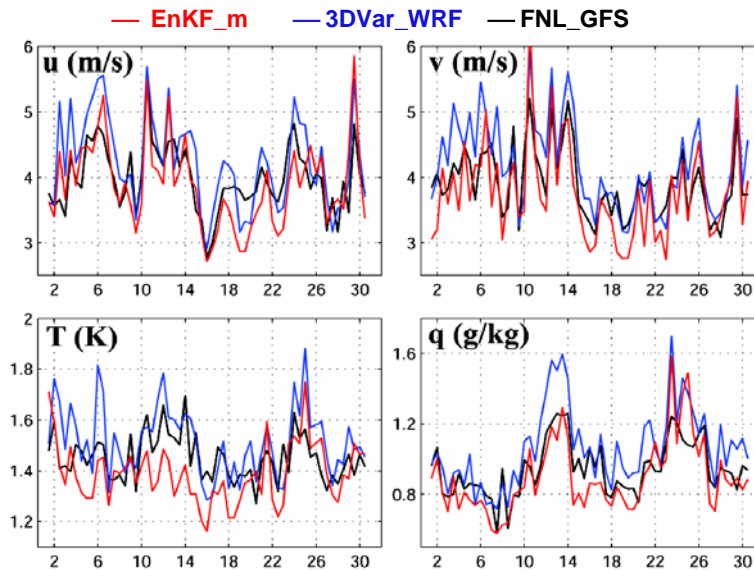
Verification: against soundings/dropsondes at standard pressure levels

Two domains with one-way nesting



(Meng and Zhang 2008a,b)

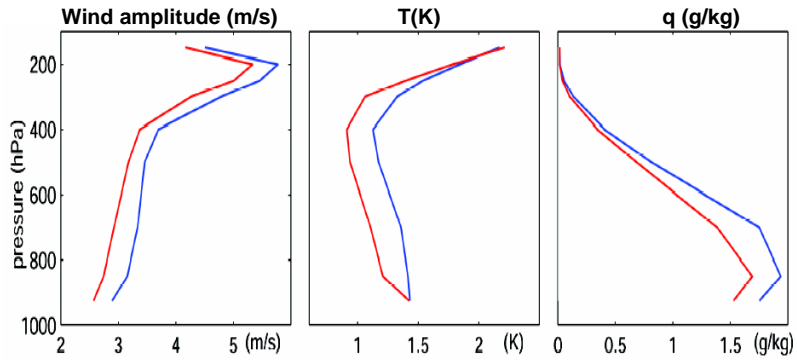
EnKF vs. 3DVar vs. FNL_GFS for June 2003: 12h fcst Prior



EnKF outperforms WRF/3DVAR as well as WRF forecast starting from FNL_GFS which assimilates many more data including satellite; FNL_GFS better than wrf-3DVar

Vertical Distribution of 12-h Forecast RMSE for June 2003

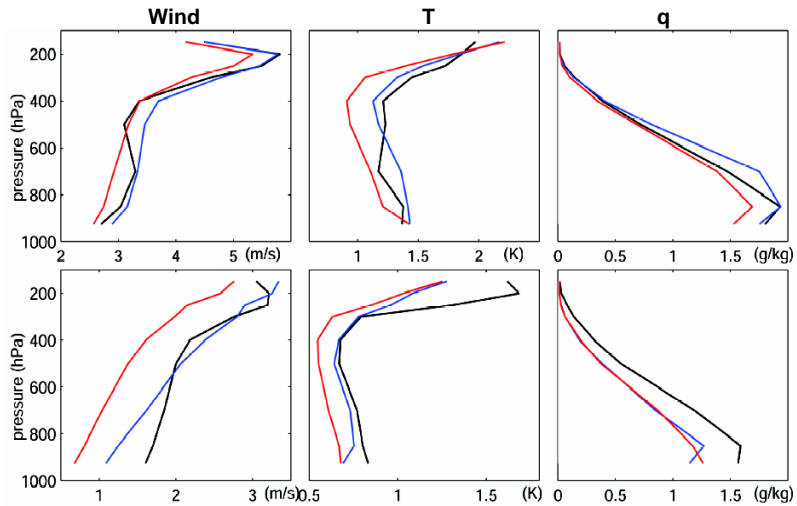
— EnKF_m — 3DVar_WRF



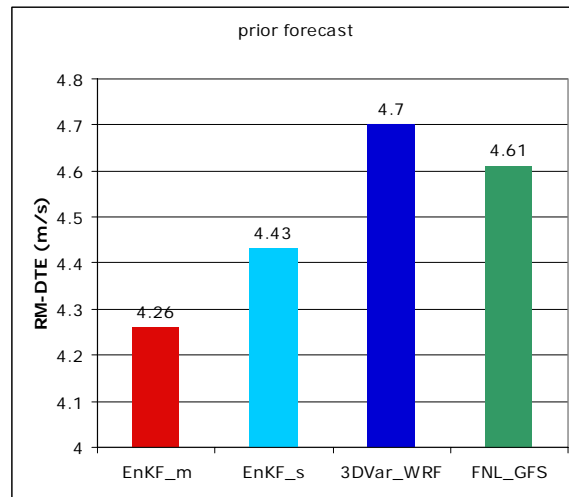
WRF-EnKF performs clearly better than WRF-3DVar in almost every vertical level

Vertical distribution of 12-h forecast (upper) and analyses (lower) RMSE

— EnKF_m — 3DVar_WRF — FNL_GFS

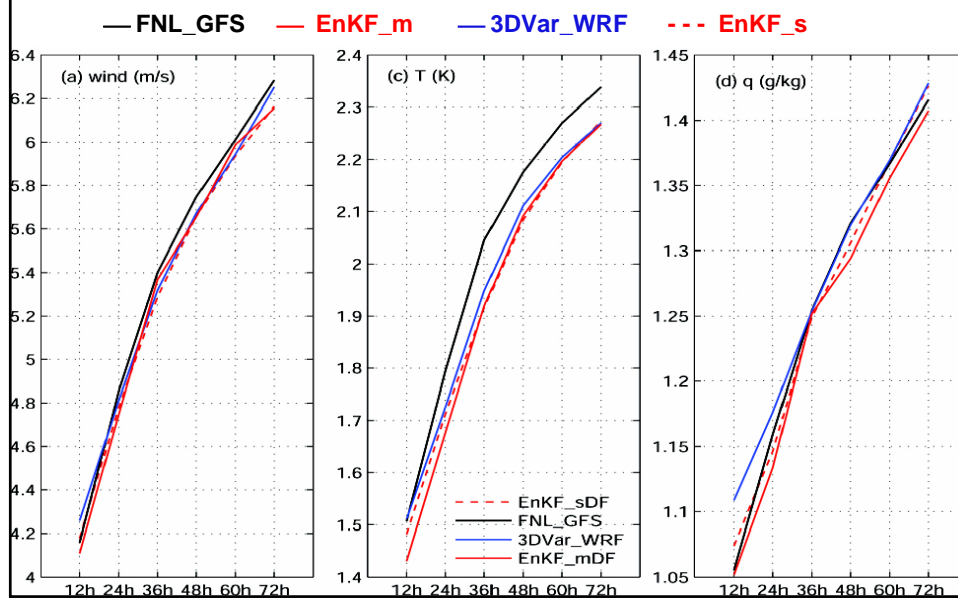


12-h forecast RM-DTE for the whole month of June 2003

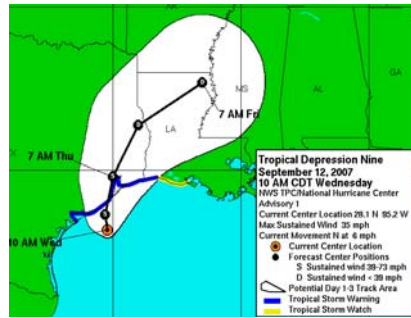
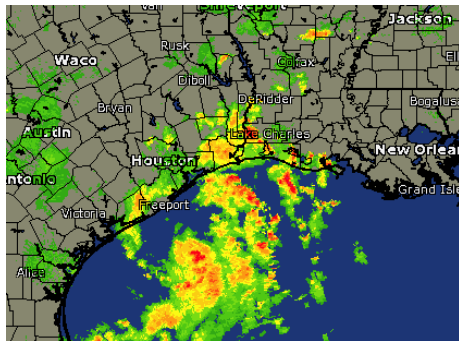


- EnKF_m has the smallest overall forecast error.
- EnKF_s has larger forecast error than EnKF_m. Both smaller than WRF-3DVar and FNL_GFS.
- FNL_GFS has smaller overall forecast error than WRF-3DVar.

Month-long 60h forecast RMSE starting from different analyses (average over 60 forecasts, twice daily)



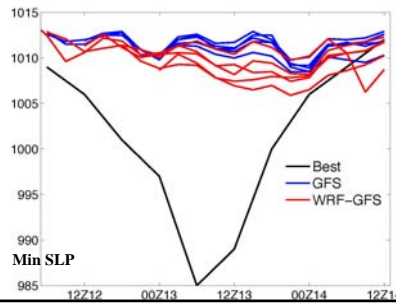
Prediction and Predictability of Hurricane Humberto (2007)



It becomes a hurricane 14hr after this NHC forecast.

Synopsis: first hurricane at TX coast since Rita (2005); fastest from first NHC warning to a category 1 hurricane; 70 million estimated property damage, 1 death

GFS (blue) & 4.5-km WRF (red) forecast: No forecast initialized with GFS FNL analysis ev 6hr from 00Z 12 to 00Z 13 predicts rapid formation



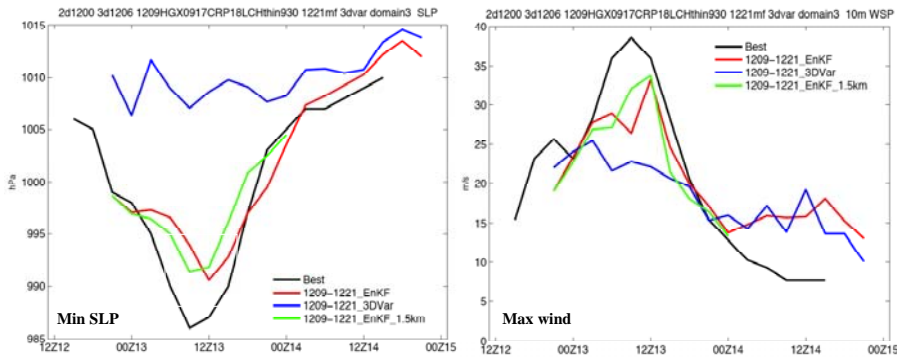
Assimilate W88D Vr for Humberto with EnKF

- **WRF domains:** D1-D2-D3 grid sizes---40.5km, 13.5km, 4.5km
 - **Physics:** WSM 6-class microphysics; YSU PBL; Grell-Devenyi CPS
- **EnKF (Meng & Zhang 2007b,c):** - 30-member ensemble
 - Initialized at 00Z 12 using 3DVar background uncertainty with FNL analysis
 - Covariance localization (Gaspari&Cohn 1999)
 - Covariance relaxation (Zhang, Snyder and Sun, 2004)

- **Data assimilated:**
 - WSR88D at KCRP, KHGX and KLCH radar radial velocity every hour from 09Z to 21Z Sept 12, 2007
 - Data assimilation are performed for all domains; obs err 3m/s
 - **Successive covariance localization:** RoI=1200km, 400km and 135km for 1/9, 1/3 and 5/9 of SOs, respectively



EnKF vs. 3Dvar for Doppler Vr Assimilation for Hurricanes



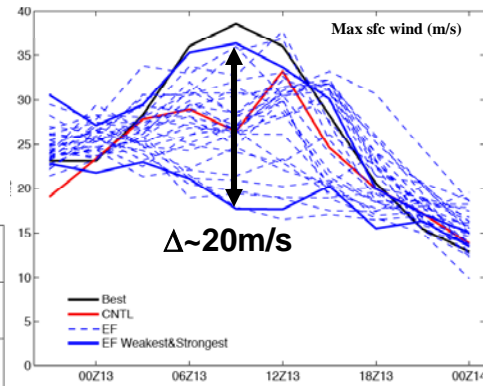
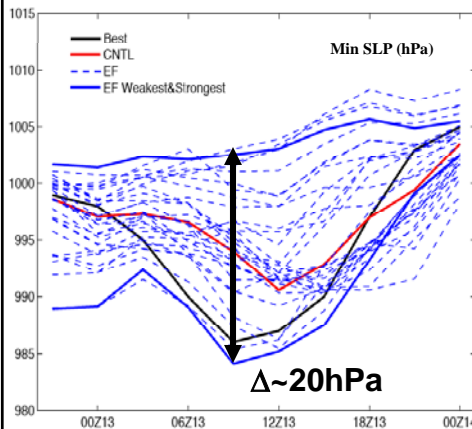
WRF single forecasts initialized with EnKF analysis at 21Z/12 captures well the rapid TC formation and deepening (red)

An additional 1.5km moving nest with same analysis even better (green)

Forecast initialized from WRF/3DVAR using the same Vr fails badly (blue).

Humberto Predictability: ensemble w/ EnKF perturbations

A 30-member WRF ensemble forecast starts at 21Z/12 with EnKF analysis + analysis uncertainties; initial ensemble perturbation realistic but small

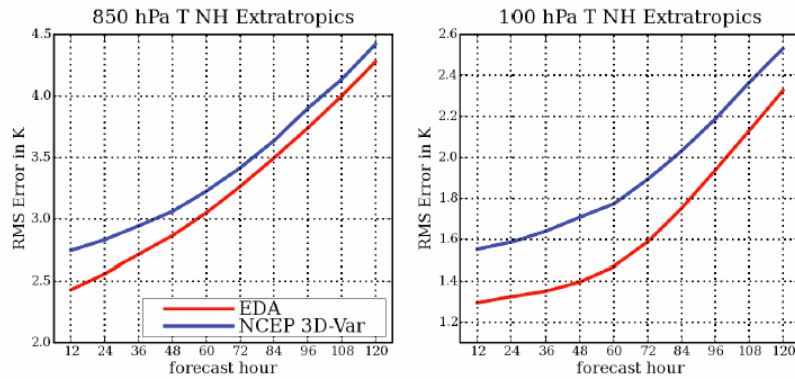


Huge ensemble spread along the subsequent ensemble forecasts maximized at the time of most intense storm in observation

Preliminary analyses show moist convection again key to the limited practical and intrinsic predictability of this fast developer

EnDA vs. 3Dvar in NCEP GFS System w/o Sat Obs

(Whitaker et al. 2008, MWR, in press)



- ✓ 6-12 hr improvement in lead time at 850, more for 100 hPa.
- ✓ improvement largest in data sparse regions (SH, stratosphere).

Comparison of 4D-Var and LETKF at JMA

T. Miyoshi and Y. Sato

N.H.

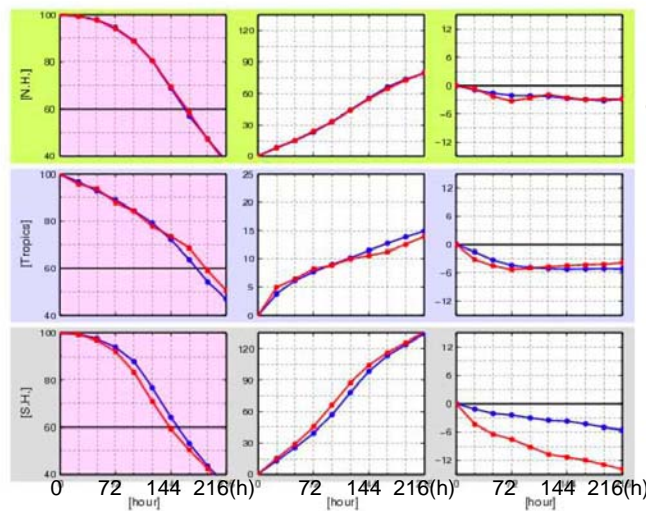
better

Tropics

better

S.H.

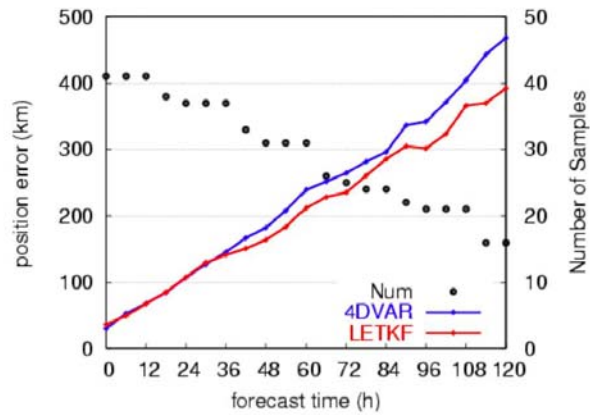
worse



Both run at operational setting w/ all data averaged over thru August 2004

RMS error against analysis

Comparison of 4-D Var and LETKF at JMA
 RMS error statistics for all typhoons in August 2004
 T. Miyoshi and Y. Sato



Operational 4D-Var

LETKF

LETKF incurs half of the computational cost of 4DVar

EnKF vs. 4DVAR: Primary Strength and Weakness

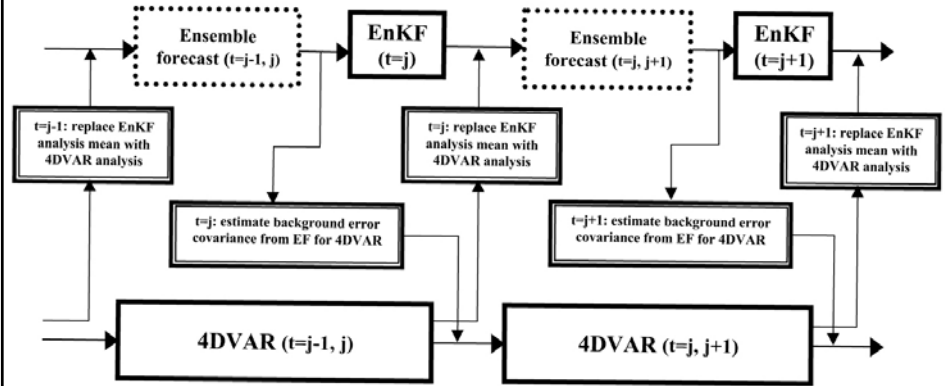
4DVAR *strength*: stronger dynamic constraint to overcome inaccurate B & first guess x_f
weakness: poor, static initial uncertainty B; single, deterministic state estimate x_a

EnKF *strength*: state-dependent B; explicit analysis uncertainty for ensemble forecasting
weakness: solely dependent on the quality of B & x_f ; more vulnerable to model error

Motivation : Coupling EnKF with 4DVAR

- Use ensemble forecast initiated from EnKF to estimate background error covariance
- Use stronger dynamics constraint of 4DVAR for deterministic state estimate

E4DVAR: Coupling EnKF with 4DVAR



E4DVAR: a prior ensemble forecast before EnKF analysis valid at t is used to estimate Pf for the subsequent 4DVAR assimilation cycle (t=j,j+1) while the 4DVAR analysis from the previous assimilation cycle (t=j-1,j) is used to replace the EnKF analysis mean for subsequent ensemble forecast.

E4DVAR1: background error covariance solely from ensemble Pf

E4DVAR2: mix static and ensemble Pf through $\mathbf{B} = \beta \mathbf{P}^f + (1 - \beta) \mathbf{B}_s$

Dynamic System and Experimental Design

$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + F, \quad i = 1, n \quad (\text{Lorenz 1996})$$

An “atmosphere-like” system on a latitude belt with chaotic behavior for suitable values of F

Truth run configuration: Degree of freedom n=80; forcing F=8.0; time step $\Delta t=0.05$ or 6 h

Forecast model used for assimilation:

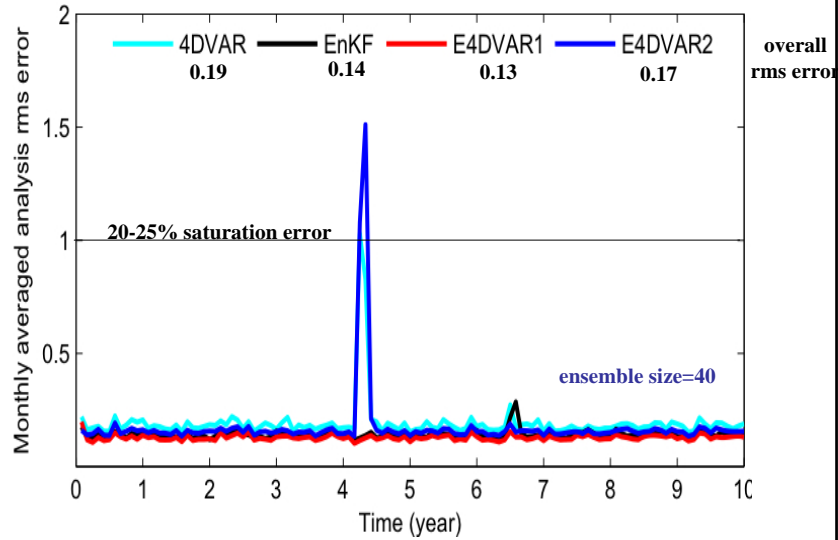
- Case 1:** Perfect-model scenario F=8.0 (no error in forecast model)
- Case 2:** Moderate model error F=8.5 (20-30% of ensemble spread over 24 h integration)
- Case 3:** Severe model error F=9.0 (35-50% of ensemble spread over 24 h integration)

Assimilation Specifics:

- Default number and frequency of observations:** 20 obs every 12 h with r.m.s. error of 0.2
- Assimilation window:** L=60h (optimum for 4DVAR in this system) or L=24h (NWP models)
- Static error covariance B:** simple diagonal matrix developed from 10-year climatology
- Number of ensemble members:** 40 (standard for most EnKF application) or 10 (low cost)
- Covariance localization (Gaspari and Cohn 1999):** r.o.i.=8 (w/o model error) and 4 (w/ model error)
- Covariance Relaxation (Zhang et al. 2004):** relaxing posterior to prior Pf $(\mathbf{x}'_i)^{new} = \alpha (\mathbf{x}'_i)^f + (1 - \alpha) (\mathbf{x}'_i)^a$

Comparison in Perfect-model Scenario (F=8.0): Default Setup

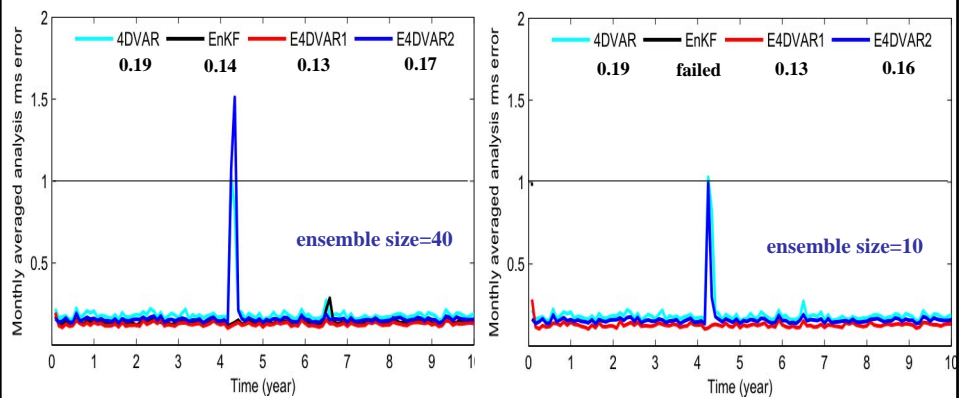
d.o.f.=80, Nobs=20, Obsfreq=12h, r.o.i.=8, $\alpha=0.5$, $\beta=0.5$, assimilation window L=60h



- With an ensemble size of 40 and no model error, all schemes performs well
- E4DVAR1 < EnKF < E4DVAR2 < 4DVAR; E4DVAR1 rms error 30% smaller than 4DVAR

Comparison in Perfect-model Scenario (F=8.0): Default Setup

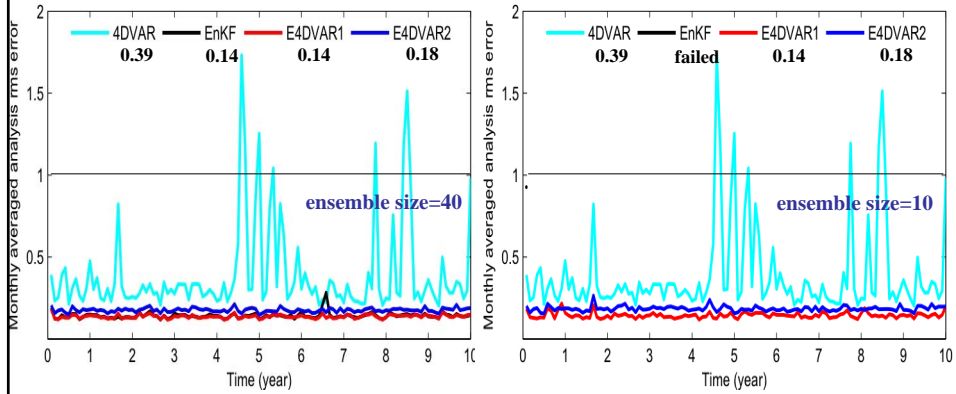
d.o.f.=80, Nobs=20, Obsfreq=12h, r.o.i.=8, $\alpha=0.5$, $\beta=0.5$, assimilation window L=60h



Both coupled schemes outperform 4DVAR even with an ensemble size of 10 while EnKF fails at small ensemble size due to sampling error and filter divergence

Comparison in Perfect-model Scenario (F=8.0): Default Setup

d.o.f.=80, Nobs=20, Obsfreq=12h, r.o.i.=8, $\alpha=0.5$, $\beta=0.5$, assimilation window L=24h

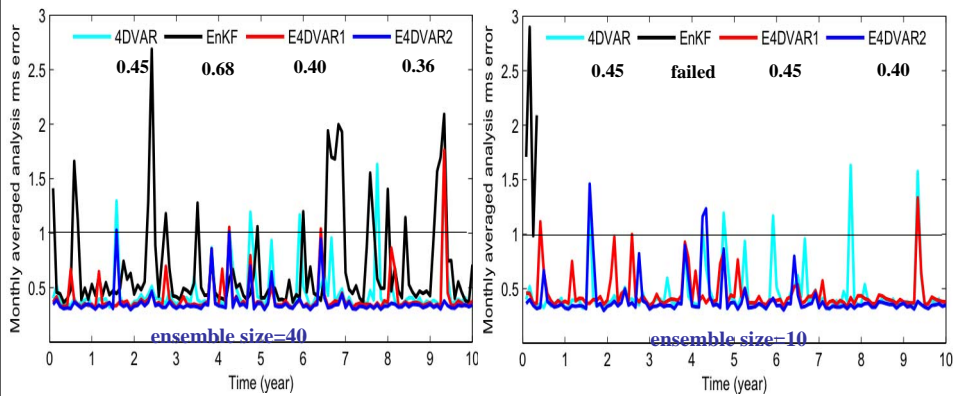


The coupled scheme is also rather insensitive to assimilation window length L but the standard 4DVAR may suffer seriously from converging to local minima when L=24h

In the perfect-model scenario, the coupling scheme w/o mixing ensemble Pf with static B has the best performance even with an ensemble size of 10

Experiments with Moderate Model Error (F=8.5): Default Setup

d.o.f.=80, Nobs=20, Obsfreq=12h, r.o.i.=4, $\alpha=0.6$, $\beta=0.5$, assimilation window L=60h

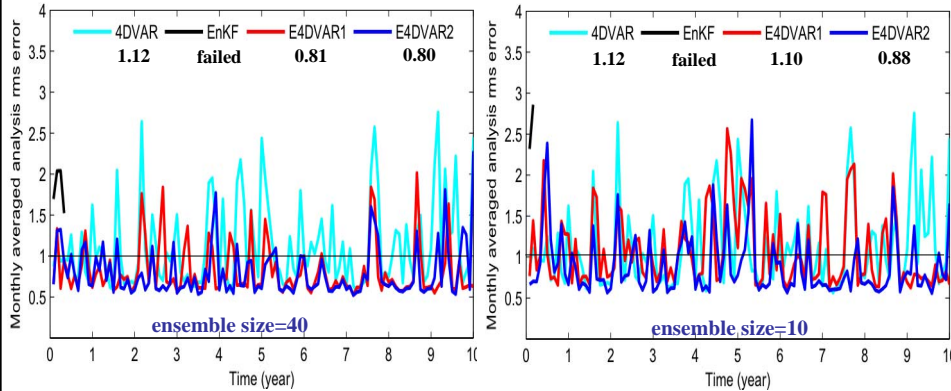


All assimilation schemes will encounter significant degradation in performance with model error but (1) 4DVAR begins to perform better than EnKF and (2) the coupled scheme which mix static B with ensemble Pf will have the best performance

Both coupled schemes outperform 4DVAR even with an ensemble size of 10 while EnKF fails at a small ensemble size due to model/sampling error and apparent filter divergence

Experiments with Severe Model Error (F=9.0): Default Setup

d.o.f.=80, Nobs=20, Obsfreq=12h, r.o.i.=4, $\alpha=0.6$, $\beta=0.5$, assimilation window L=60h



With severe model error, both coupled schemes have marginally acceptable performance while EnKF fails even with an ensemble size of 40 and 4DVAR performance becomes marginally unacceptable with many more frequent local minima

E4DVAR1 will become marginally unacceptable with an ensemble size of 10 and default setup while E4DVAR2 with hybrid B performs marginally acceptable; both will be acceptable if setup is optimally tuned even with 10 ensemble members

Overview of Experiments with Assimilation Window L=60h

		Ensemble size m = 40, default parameter setup		Ensemble size m = 40, tuned parameter setup		Ensemble size m = 10, default parameter setup		Ensemble size m = 10, tuned parameter setup	
		analysis error	default R, α β	analysis error	tuned R, α β	analysis error	default R, α β	analysis error	tuned R, α β
Perfect model F = 8.0	4DVAR	0.19	NA	0.19	NA	0.19	NA	0.19	NA
	EnKF	0.14	8, 0.5, NA	0.12	12, 0.3, NA	Failed	8, 0.5, NA	0.84	4, 0.7, NA
	E4DVAR1	0.13	8, 0.5, 1.0	0.11	12, 0.3, 1.0	0.13	8, 0.5, 1.0	0.13	8, 0.5, 1.0
	E4DVAR2	0.17	8, 0.5, 0.5	0.11	12, 0.3, 1.0	0.16	8, 0.5, 0.5	0.13	8, 0.5, 1.0
Moderate model error F = 8.5	4DVAR	0.45	NA	0.45	NA	0.45	NA	0.45	NA
	EnKF	0.68	4, 0.6, NA	0.64	3, 0.6, NA	Failed	4, 0.6, NA	1.48	3, 0.7, NA
	E4DVAR1	0.40	4, 0.6, 1.0	0.38	3, 0.6, 1.0	0.45	4, 0.6, 1.0	0.38	4, 0.7, 1.0
	E4DVAR2	0.36	4, 0.6, 0.5	0.35	3, 0.6, 0.4	0.40	4, 0.6, 0.5	0.36	4, 0.7, 0.3
Severe model error F = 9.0	4DVAR	1.12	NA	1.12	NA	1.12	NA	1.12	NA
	EnKF	Failed	4, 0.6, NA	1.24	3, 0.6, NA	Failed	4, 0.6, NA	1.76	2, 0.6, NA
	E4DVAR1	0.81	4, 0.6, 1.0	0.70	3, 0.6, 1.0	1.10	4, 0.6, 1.0	0.70	3, 0.7, 1.0
	E4DVAR2	0.80	4, 0.6, 0.5	0.66	3, 0.6, 0.4	0.88	4, 0.6, 0.5	0.68	4, 0.7, 0.3

R: localization radius of influence or r.o.i.; **α :** relaxation coefficient; **β :** mixing coefficient

$$(\mathbf{x}_i')^{new} = \alpha(\mathbf{x}_i')^f + (1 - \alpha)(\mathbf{x}_i')^a$$

$$\mathbf{B} = \beta\mathbf{P}^f + (1 - \beta)\mathbf{B}_s$$

Concluding Remarks

E4DVAR coupling: EnKF for estimating background error covariance through ensemble forecast; 4DVAR for deterministic analysis through variational minimization

Coupling strength: E4DVAR benefits from using the state-dependent uncertainty provided by ensemble-based filters EnKF while taking advantage of using of stronger dynamic constraints in 4DVAR in preventing filter divergence; E4DVAR is rather insensitive to assimilation window length and ensemble size

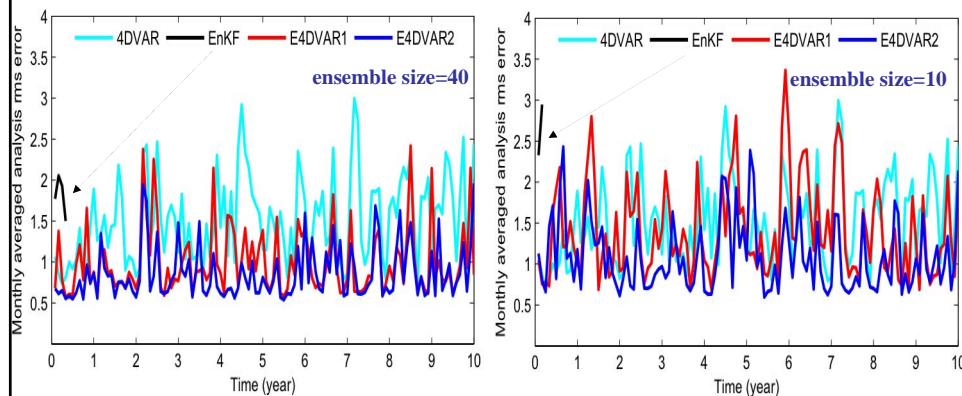
Coupling performance: E4DVAR outperforms both 4DVAR and EnKF under both perfect- and imperfect-model scenarios though degrades significantly with model error

Coupling cost: E4DVAR performs well even an ensemble size of 10; only very small computational cost addition to 4DVAR; further reduction of cost is likely in complex systems using EnKF analysis as first guess for minimization and pre-conditioning

Future outlook: there should be no war between EnKF and 4DVAR; they could well work with each other; further tests in complex models are in demand and in planning

Experiments with Severe Model Error ($F=9.0$): Default Setup

d.o.f.=80, Nobs=20, Obsfreq=12h, r.o.i=4, $\alpha=0.6$, $\beta=0.5$, **assimilation window $L=24h$**

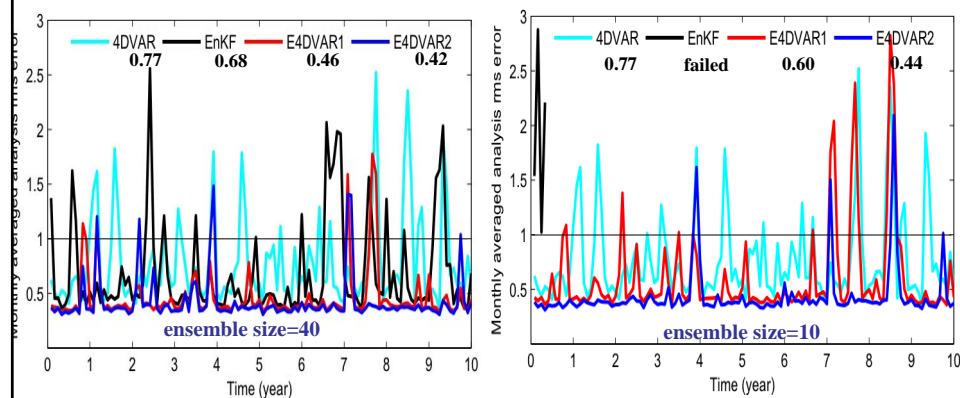


Summary of Experiments with Assimilation Window $L=24h$

		Ensemble size $m = 40$, default parameter setup		Ensemble size $m = 40$, tuned parameter setup		Ensemble size $m = 10$, default parameter setup		Ensemble size $m = 10$, tuned parameter setup	
		analysis error	default R, α, β	analysis error	tuned R, α, β	analysis error	default R, α, β	analysis error	tuned R, α, β
Perfect model $F = 8.0$	4DVAR	0.39	NA	0.39	NA	0.39	NA	0.39	NA
	EnKF	0.14	8, 0.5, NA	0.12	12, 0.3, N	Failed	8, 0.5, NA	0.84	4, 0.7, NA
	E4DVAR1	0.14	8, 0.5, 1.0	0.12	12, 0.3, 0.8	0.14	8, 0.5, 1.0	0.14	8, 0.5, 1.0
	E4DVAR2	0.18	8, 0.5, 0.5	0.15	12, 0.3, 0.8	0.18	8, 0.5, 0.5	0.16	8, 0.5, 0.8
Moderate model error $F = 8.5$	4DVAR	0.77	NA	0.77	NA	0.77	NA	0.77	NA
	EnKF	0.68	4, 0.6, NA	0.64	3, 0.6, NA	Failed	4, 0.6, NA	1.48	3, 0.7, NA
	E4DVAR1	0.46	4, 0.6, 1.0	0.46	4, 0.6, 1.0	0.60	4, 0.6, 1.0	0.52	3, 0.5, 1.0
	E4DVAR2	0.42	4, 0.6, 0.5	0.41	3, 0.5, 0.4	0.44	4, 0.6, 0.5	0.42	4, 0.6, 0.3
Severe model error $F = 9.0$	4DVAR	1.52	NA	1.52	NA	1.52	NA	1.52	NA
	EnKF	Failed	4, 0.6, NA	1.23	3, 0.6, NA	Failed	4, 0.6, NA	1.74	2, 0.6, NA
	E4DVAR1	1.00	4, 0.6, 1.0	1.00	4, 0.6, 1.0	1.41	4, 0.6, 1.0	1.39	4, 0.7, 1.0
	E4DVAR2	0.86	4, 0.6, 0.5	0.86	4, 0.6, 0.5	1.09	4, 0.6, 0.5	1.01	4, 0.6, 0.3

Experiments with Moderate Model Error ($F=8.5$): Default Setup

d.o.f.=80, Nobs=20, Obsfreq=12h, r.o.i.=4, $\alpha=0.6$, $\beta=0.5$, assimilation window $L=24h$



The coupled scheme is less sensitive to assimilation window length L but the standard 4DVAR may suffer seriously from converging to local minima when $L=24h$

In the imperfect-model scenario, the coupling scheme E4DVAR2 that mixes ensemble Pf with static B has the best performance