A Local Ensemble Transform Kalman Filter:
Perfect Model Results with the Lorenz-95 Model

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Thanks to: J. S. McDonnell Foundation,

NOAA/THORPEX, NSF

Local Ensemble Transform Kalman Filter

- LETKF is a model-independent data assimilation method designed for spatiotemporally chaotic systems that is:
  - Computationally efficient and robust
  - Mathematically simple and flexible
- This approach combines elements of the Ensemble Transform Kalman Filter [Bishop et al. 2001] and the Local Ensemble Kalman Filter [Ott et al. 2004].

Starting Point

- Data assimilation by statistical interpolation [Lorenc 1981].
- Given a background (first-guess) model state x^b with (presumed) covariance B, observations y^o with covariance R, and a function H from model space to observation space, we seek to minimize the function

$$J(x) = (x - x^b)^T B^{-1} (x - x^b) + [y^o - H(x)]^T R^{-1} [y^o - H(x)].$$

- The minimizing state x^a is called the analysis.

Ensemble Kalman Filtering

- Ensemble Kalman filters [Evensen 1994] evolve an ensemble of initial conditions with the nonlinear model and let x^b and B be the sample mean and covariance of the ensemble forecast states at the analysis time.
- Bad news: The background covariance B reflects only uncertainties in the space S spanned by the ensemble.
- Good news: The analysis takes place in the low-dimensional space S (computationally efficient).

Guiding Question

- Which linear combination of the ensemble states best fits the data?
- Let k be the number of ensemble members, and let X be a matrix of normalized ensemble perturbations: each column of X is the difference between an ensemble state x^b_j and the ensemble mean x^b, divided by $\sqrt{k - 1}$ so that $B = XX^T$.
- Express a model state x in S as $x = x^b + Xw$, where w is a k-dimensional weight vector. Which w is best?

Linearization of $H(x)$ in Ensemble Space S

- For each ensemble state x^b_j, let $y^b_j = H(x^b_j)$. Let the mean of this background observation ensemble be y^b, and let Y be the matrix with columns $(y^b_j - y^b)/\sqrt{k - 1}$.

- Make the linear approximation $H(x^b + Xw) \approx y^b + Yw$.

- In terms of w, the function to be minimized is

$$J(w) \approx w^T w + (y^o - y^b - Yw)^T R^{-1} (y^o - y^b - Yw).$$

(The background mean of w is 0 and its background covariance is the identity matrix.)

Analysis Mean and Covariance

- In the w coordinate system, the analysis mean w^a and covariance A are given by the standard Kalman filter equations

$$A = (I + Y^T R^{-1} Y)^{-1}$$
$$w^a = A Y^T R^{-1} (y^o - y^b)$$

- The matrix that is inverted to find A is small (k by k) and has no small eigenvalues.

- To perform multiplicative covariance inflation, divide I by $r > 1$ in the formula for A.

Analysis Ensemble

- To form the analysis ensemble weight vectors w^a_j, add to w^a the columns of the symmetric matrix $W = [(k - 1)A]^{1/2}$; this ensures the correct analysis covariance.
- Any matrix for which $WW^T = (k - 1)A$ would do; this is the choice available in a square root filter [Tippett et al. 2003].
- Our choice minimizes the distance between the background and analysis ensembles and ensures that, when done locally, the analysis ensemble varies continuously from one region to the next [Ott et al. 2004].

Localization

- For an ensemble of moderate size ($k < 100$), the linear combination that best fits the data in one region may be significantly different from the best linear combination in another region.

- Localize by doing a separate analysis at each model grid point, ignoring data beyond a certain distance [Houtekamer & Mitchell 1998].

- One can choose which data to use, and do the resulting analysis, independently at each grid point.

Perfect Model Results

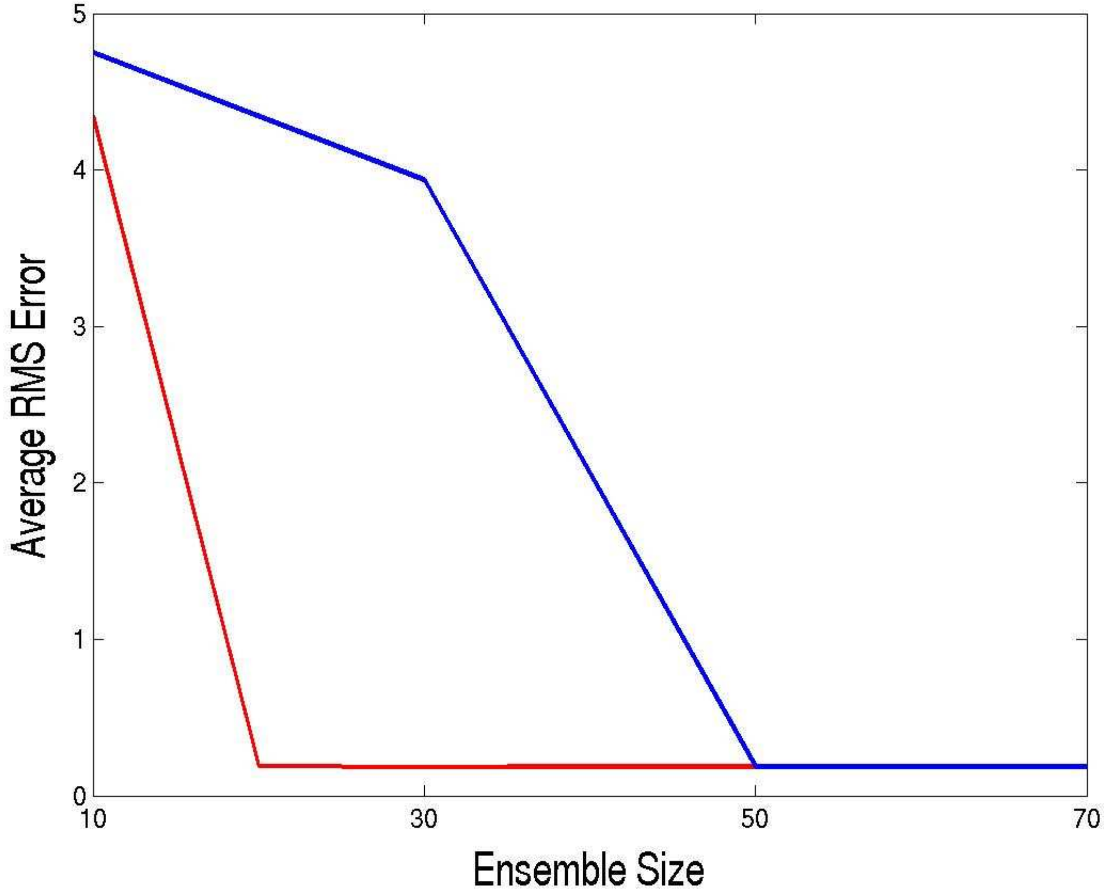
- These results were obtained by John Harlim using the model proposed by Lorenz in 1995:

$$dx_j/dt = (x_{j+1} - x_{j+2})x_{j-1} - x_j + 8$$

for $j = 1, 2, \ldots N$.

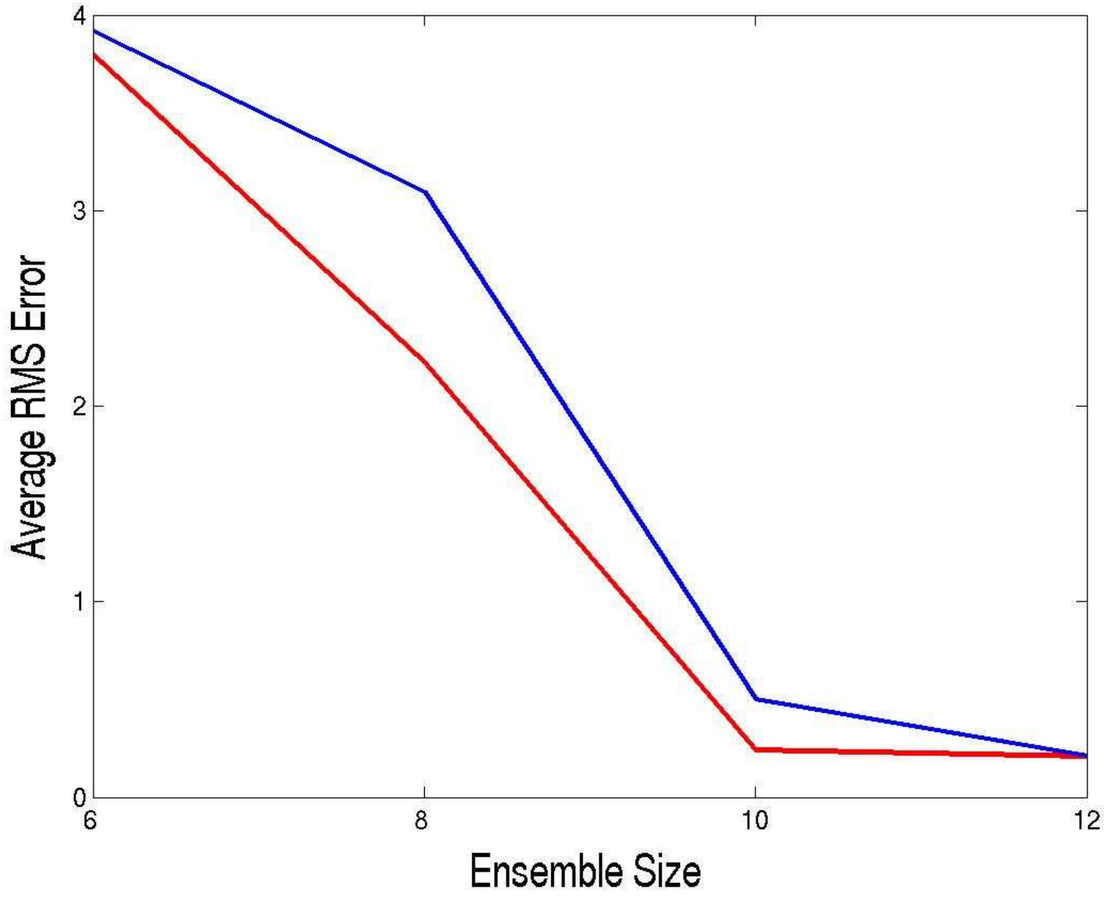
- From a “truth run” we generated “observations” at each node with $R = I$.
- We measured the RMS error of our method for various ensemble sizes and localizations, using covariance inflation $r = 1.04$.

Global Filter



Red: N = 40, Blue: N = 80

Local Filter



Using only observations from within 6 grid points

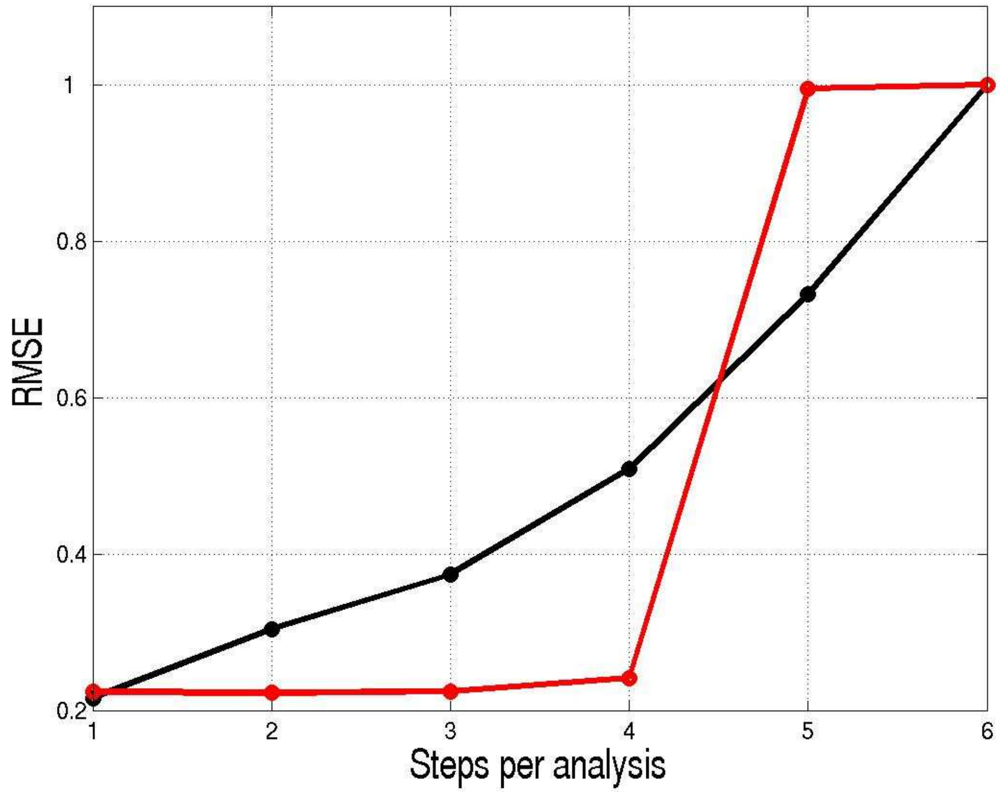
Asynchronous Observations

- In an operational setting, data cannot be assimilated as frequently as it is taken; several hours worth of data is assimilated at one analysis time.

- Which linear combination of the ensemble trajectories best fits the data?

- For observations taken at time t, apply H to the ensemble states at time t when forming the background observation ensemble vectors y^b_j, and proceed exactly as before. This simplifies the 4D approach described in [Hunt et al. 2004].

4DLETKF Results



Black: LETKF ignoring intermediate observations

Red: 4DLETKF with r = 1.12

Conclusions

- LETKF scales well to large systems (similar results up to $N = 400$ and we presume beyond).
- It does not require explicitly linearizing the observation operator.
- The amount of localization is easily adjusted.
- Observations taken at different times can be assimilated simultaneously.
- More from our group:

  `http://keck2.umd.edu/weather/`